(b)	$\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=2\sec^2 3\theta-8\Rightarrow\tan 3\theta=2\sec^2 3\theta-8$ $\Rightarrow\tan 3\theta=2\left(1+\tan^2 3\theta\right)-8$	M1
	$\Rightarrow 2\tan^2 3\theta-\tan 3\theta-6=0$ or e.g. $\Rightarrow 2\tan^2 3\theta-\tan 3\theta=6$	A1
	$\left(2\tan 3\theta+3\right)\left(\tan 3\theta-2\right)=0\Rightarrow\tan 3\theta=-\dfrac{3}{2},\ 2$ $\tan 3\theta=-\dfrac{3}{2}\Rightarrow 3\theta=...\Rightarrow\theta=...$ or $\tan 3\theta=2\Rightarrow 3\theta=...\Rightarrow\theta=...$	dM1
	$\theta=0.37,\ 0.72,\ 1.42$	A1A1
		(5)
		Total 8

Allow if a different variable used (such as *x*).
For mixed variables allow the M's but only allow the first A
(and final A's) if recovered or interpreted correctly as the same variable.

M1: Writes the lhs as $k\tan 3\theta$ and applies $\sec^2 3\theta=\pm1\pm\tan^2 3\theta$ on the rhs.
A1: Reaches the correct 3TQ in $\tan 3\theta$ with terms collected but not necessarily all on one side.
dM1: Solves their 3TQ in $\tan 3\theta$ by any method (usual rules) including a calculator (you may need to check) to obtain at least one value for $\tan 3\theta$ and uses correct processing to obtain at least one value for θ e.g. finds inverse tan and divides by 3.
A1: Awrt at least one of these answers. Must be in radians and must come from the correct 3TQ.
A1: Awrt these answers and no extras in range. Must be in radians.

Note that there may be more convoluted methods which include writing the lhs as $k\dfrac{\sin 3\theta}{\cos 3\theta}$ e.g.

$$\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=2\sec^2 3\theta-8\Rightarrow\frac{\sin 3\theta}{\cos 3\theta}=2\sec^2 3\theta-8=\frac{2}{\cos^2 3\theta}-8$$

$$\Rightarrow\sin 3\theta\cos 3\theta=2-8\cos^2 3\theta\Rightarrow\frac{1}{2}\sin 6\theta=2-8\left(\frac{\cos 6\theta+1}{2}\right)$$

$$\Rightarrow\sin 6\theta+8\cos 6\theta=-4$$

$$\Rightarrow\sqrt{65}\sin\left(6\theta+1.4464\right)=-4\Rightarrow\sin\left(6\theta+1.4464\right)=-\frac{4}{\sqrt{65}}\Rightarrow\theta=...$$

$$\theta=0.37,\ 0.72,\ 1.42$$

In such cases, the first M mark will only be scored when they reach an equation they can solve directly e.g. in the above example when they reach $\sqrt{65}\sin\left(6\theta+1.4464\right)=-4$

If you are in doubt whether an approach deserves any credit use Review.

Question Number	Scheme	Marks
9(a)	$x=0 \Rightarrow \frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)=0 \Rightarrow 3y+\frac{\pi}{4}=\pi,\ 2\pi,... \Rightarrow y=...$	M1
	$y=\frac{\pi}{4}$ or $y=\frac{7\pi}{12}$	A1
	$y=\frac{\pi}{4}$ and $y=\frac{7\pi}{12}$	A1
		(3)

Condone any confusion about which point is which and just look for the correct values.
Look for answers given in the body of the question or on Figure 2

M1: Correct strategy to find y at A or B.

Requires an attempt to solve either $3y+\frac{\pi}{4}=\pi$ or $3y+\frac{\pi}{4}=2\pi$

May be implied by at least one correct value and may be seen embedded e.g. $\left(0,\ \frac{\pi}{4}\right), \left(0,\ \frac{7\pi}{12}\right)$

Condone e.g. $\left(\frac{\pi}{4},\ 0\right), \left(\frac{7\pi}{12},\ 0\right)$

Allow equivalent work e.g. $3y+\frac{\pi}{4}=0 \Rightarrow y=-\frac{\pi}{12} \Rightarrow y=-\frac{\pi}{12}+\frac{\pi}{3}, -\frac{\pi}{12}+\frac{2\pi}{3}$

May be implied by decimal answers e.g. awrt 0.79 or awrt 1.8

A1: One correct value. Allow simplified or unsimplified and may be seen embedded e.g. $\left(0,\ \frac{\pi}{4}\right), \left(0,\ \frac{7\pi}{12}\right)$

Condone unsimplified for this mark e.g. $y=\frac{1}{3}\left(\pi-\frac{\pi}{4}\right)$ for $\frac{\pi}{4}$ or e.g. $y=\frac{2\pi}{3}-\frac{\pi}{12}$ for $\frac{7\pi}{12}$

A1: Both correct and simplified and may be seen embedded e.g. $\left(0,\ \frac{\pi}{4}\right), \left(0,\ \frac{7\pi}{12}\right)$

Condone e.g. $\left(\frac{\pi}{4},\ 0\right), \left(\frac{7\pi}{12},\ 0\right)$ and condone unsimplified as long as they are single fractions e.g. $\frac{2\pi}{8}$

Alternative:

$$\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)=0 \Rightarrow \frac{2}{3}\sin 3y\cos\frac{\pi}{4}+\frac{2}{3}\cos 3y\sin\frac{\pi}{4}=0 \Rightarrow \tan 3y=-1 \Rightarrow 3y=\frac{3\pi}{4},\ \frac{7\pi}{4}$$

Score M1 for an attempt to solve either $3y=\frac{3\pi}{4}$ or $3y=\frac{7\pi}{4}$ and then A1A1 as above.

(b)	$x=\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{2}{3}\times3\cos\left(3y+\frac{\pi}{4}\right)\left(=2\cos\left(3y+\frac{\pi}{4}\right)\right)$	B1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\cos\left(3y+\frac{\pi}{4}\right)}\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{1}{4\cos^2\left(3y+\frac{\pi}{4}\right)}=\frac{1}{4\left(1-\sin^2\left(3y+\frac{\pi}{4}\right)\right)}$	M1
	$=\frac{1}{4\left(1-\left(\frac{3x}{2}\right)^2\right)}$	**d**M1
	$=\frac{1}{4-9x^2}$	A1
		(4)

B1: Correct $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form.

Do not condone missing brackets e.g. $2\cos 3y+\frac{\pi}{4}$ unless recovered or implied by subsequent work.

M1: For **any 2** of the following applied to their $\frac{\mathrm{d}x}{\mathrm{d}y}$:

- finds reciprocal (may have already happened if they find $\frac{\mathrm{d}y}{\mathrm{d}x}$)
- squares
- applies $\cos^2\left(3y+\frac{\pi}{4}\right)=1-\sin^2\left(3y+\frac{\pi}{4}\right)$

Note that this mark (and the next one) may be implied by equivalent work e.g.

$x=\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)\Rightarrow\sin\left(3y+\frac{\pi}{4}\right)=\frac{3x}{2}\Rightarrow\cos\left(3y+\frac{\pi}{4}\right)=\frac{\sqrt{4-9x^2}}{2}$

$\frac{\mathrm{d}x}{\mathrm{d}y}=2\cos\left(3y+\frac{\pi}{4}\right)=\sqrt{4-9x^2}\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\frac{1}{4-9x^2}$

dM1: Uses all 3 of the above correctly to obtain $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ in terms of *x*.

A1: Correct answer.

You can condone e.g. missing trailing brackets within their working e.g. $\frac{1}{4(1-\sin^2\left(3y+\frac{\pi}{4}\right)}$

Special case: allow this mark if the only error in their solution is to obtain $\frac{\mathrm{d}x}{\mathrm{d}y}=-\frac{2}{3}\times3\cos\left(3y+\frac{\pi}{4}\right)$

(b) Way 2	$x=\frac{2}{3}\sin\left(3y+\frac{\pi}{4}\right)\Rightarrow 3y+\frac{\pi}{4}=\sin^{-1}\frac{3x}{2}$	B1
	$3y+\frac{\pi}{4}=\sin^{-1}\frac{3x}{2}\Rightarrow 3\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{1-\frac{9x^2}{4}}}\times\frac{3}{2}$	M1
	$3\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{1-\frac{9x^2}{4}}}\times\frac{3}{2}\Rightarrow\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=\left(\frac{1}{2\sqrt{1-\frac{9x^2}{4}}}\right)^2$	**d**M1
	$=\frac{1}{4-9x^2}$	A1

Way 2:

B1: Correct equation in any form connecting y and $\sin^{-1}\frac{3x}{2}$.

M1: Differentiates to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{k}{\sqrt{1-\left(\frac{3x}{2}\right)^2}}$ or equivalent. Condone missing brackets around $\frac{3x}{2}$.

dM1: Squares and makes $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ the subject.

A1: Correct answer.

(c)	Identifies that one of the gradients is $\sqrt{"4"}$ or that one of the gradients is $\frac{1}{\sqrt{"4"}}$ **or** Identifies that one of the gradients is 2 or that one of the gradients is $\frac{1}{2}$	B1ft
	At A: $y-"\frac{\pi}{4}"="2"x$ **and** At B: $y-"\frac{7\pi}{12}"="\frac{1}{2}"x$	M1
	$"2"x+"\frac{\pi}{4}"="\frac{1}{2}"x+"\frac{7\pi}{12}"\Rightarrow x=...$	**d**M1
	$x=\frac{2\pi}{9}$	A1
		(4)
		Total 11

Note that in this part the question says to use the answer to part (b) i.e. their $\frac{1}{p-qx^2}$

If they use their $\frac{1}{p-qx^2}$ then the ft is available as shown.

If candidates "start again" or e.g. use their $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of *y* with their *y* values from part (a) then the value(s) must be <u>correct</u>.

B1ft: Using the answer to part (b):

Award this mark for **one** gradient identified as $\sqrt{p}$ **or** $\frac{1}{\sqrt{p}}$ where p is their value from part (b).

or not using the answer to part (b):

One gradient identified as 2 **or** $\frac{1}{2}$

M1: Correct straight line method for the equation of the normal at *A* **and** the tangent at *B*.
They must have

- 2 values for *y* from part (a) (ignore how these were labelled)
- both gradients **positive**

For the **normal at *A*** this requires a gradient of $\sqrt{p}$ or 2 with $x = 0$ and their smaller value of *y* from part (a)
If they use $y = mx + c$ they must proceed as far as $c = \ldots$

For the **tangent at *B*** this requires a gradient of $\frac{1}{\sqrt{p}}$ or $\frac{1}{2}$ with $x = 0$ and their larger value of *y* from part (a)
If they use $y = mx + c$ they must proceed as far as $c = \ldots$

dM1: Correct method for both lines as defined above and solves simultaneously (may be via a calculator) to find the *x* coordinate of *D*.
Depends on the previous mark.

A1: Cso. Correct value $x=\frac{2\pi}{9}$.
Ignore any attempts to find the *y* coordinate.
If they use part (b) then full marks can be awarded following an incorrect value for *q* provided all the other work is correct.