(c)	e.g. $\mathrm{f}(x)=x-\frac{1}{3}\arctan\left(\frac{3}{2x}\right)$ $\mathrm{f}(0.4305)=0.4305-\frac{1}{3}\arctan\left(\frac{3}{2\times0.4305}\right)\left(=6.38\times10^{-5}\right)$ $\mathrm{f}(0.4295)=0.4295-\frac{1}{3}\arctan\left(\frac{3}{2\times0.4295}\right)\left(=-1.141...\times10^{-3}\right)$ or e.g. $\mathrm{f}(x)=3\mathrm{e}^{-x^2}\cos3x-2x\mathrm{e}^{-x^2}\sin3x$ $\mathrm{f}(0.4305)=3\mathrm{e}^{-(0.4305)^2}\cos3(0.4305)-2(0.4305)\mathrm{e}^{-(0.4305)^2}\sin3\,(0.4305)\left(=-4.968...\times10^{-4}\right)$ $\mathrm{f}(0.4295)=3\mathrm{e}^{-(0.4295)^2}\cos3(0.4295)-2(0.4295)\mathrm{e}^{-(0.4295)^2}\sin3\,(0.4295)\left(=8.885...\times10^{-3}\right)$	M1
	Sign change therefore x is 0.430 to 3dp	A1
		(2)
		Total 9

(c)

M1: Chooses a suitable function and attempts its value at both 0.4305 and 0.4295

For the attempt we need to see embedded values as in scheme or one value correct to 1sf for their suitable function.

The function they are using must be seen or implied by embedded values.

Allowable functions are:

- $x-\frac{1}{3}\arctan\left(\frac{3}{2x}\right)$ or multiples of this
- $3\mathrm{e}^{-x^2}\cos3x-2x\mathrm{e}^{-x^2}\sin3x$ or multiples of this following through on their $\frac{\mathrm{d}y}{\mathrm{d}x}$
- $3\cos3x-2x\sin3x$ or multiples of this following through on their $\frac{\mathrm{d}y}{\mathrm{d}x}$

Unlikely, but it is also acceptable to pick a tighter interval e.g. 0.4305 and 0.4301

A1: Requires correct differentiation (if $\frac{\mathrm{d}y}{\mathrm{d}x}$ is used)

- both values correct (rounded or truncated to 1sf)
- a valid reason that includes a reference to the sign change e.g. < 0, > 0
- a minimal conclusion which could be ✓, QED, root

Note we condone the omission of a reference to continuity.

Attempts to use repeated iteration score no marks.

Question Number	Scheme	Marks
8(a)	$\tan 3x \equiv \tan(2x+x) \equiv \dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$ or e.g. $\tan 3x \equiv \dfrac{\sin 3x}{\cos 3x} \equiv \dfrac{\sin(2x+x)}{\cos(2x+x)} \equiv \dfrac{\sin 2x\cos x+\cos 2x\sin x}{\cos 2x\cos x-\sin 2x\sin x} \equiv \dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$	M1
	$\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x} \equiv \dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}$	dM1
	$\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x} \equiv \dfrac{2\tan x+\tan x-\tan^3 x}{1-\tan^2 x-2\tan^2 x}$ $\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$ *	A1*
		(3)

Condone use of a different variable e.g. θ for x.
Condone the use of mixed variables for the M marks as long as the intention is clear.
Condone clear use of "tan" written differently e.g. "tg"

M1: Writes $\tan 3x$ as $\tan(2x + x)$ which may be implied and applies the addition formula for $\tan(A + B)$ to obtain an expression in terms of $\tan x$ and $\tan 2x$ only.
Condone one sign slip only.

May also reach this point by writing $\tan 3x$ as $\dfrac{\sin 3x}{\cos 3x}$ and then uses the addition formulae for $\sin(A + B)$ and $\cos(A + B)$ to obtain an expression in terms of $\tan x$ and $\tan 2x$ only.
Condone one sign slip only.

dM1: Replaces $\tan 2x$ with $\dfrac{2\tan x}{1-\tan^2 x}$ to express $\tan 3x$ in terms of $\tan x$ only.

A1*: Reaches the right hand side with sufficient working shown.
For this mark condone minor notational errors (e.g. a missing x) but not consistent poor notation throughout such as mixed variables or writing $\tan^2 x$ as $\tan x^2$

Attempts to use higher multiples of x e.g. $\tan 3x \equiv \tan(4x - x)$ should be sent to review.

Way 2	$\tan 3x \equiv \dfrac{\sin 3x}{\cos 3x} \equiv \dfrac{\sin(2x+x)}{\cos(2x+x)} \equiv \dfrac{\sin 2x\cos x+\cos 2x\sin x}{\cos 2x\cos x-\sin 2x\sin x}$ $\equiv \dfrac{2\sin x\cos^2 x+(\cos^2 x-\sin^2 x)\sin x}{(\cos^2 x-\sin^2 x)\cos x-2\sin^2 x\cos x}$	M1
	$\equiv \dfrac{2\sin x\cos^2 x+\cos^2 x\sin x-\sin^3 x}{\cos^3 x-\sin^2 x\cos x-2\sin^2 x\cos x} \equiv \dfrac{3\sin x\cos^2 x-\sin^3 x}{\cos^3 x-3\sin^2 x\cos x}$	dM1
	$\equiv \dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$ *	A1*

Condone use of a different variable e.g. θ for x.

Condone the use of mixed variables for the M marks as long as the intention is clear.

M1: Writes $\tan 3x$ as $\dfrac{\sin 3x}{\cos 3x}$ and then uses the addition formulae for sin (A + B) and cos(A + B) condoning one sign slip and then applies $\sin 2x = 2\sin x\cos x$ and $\cos 2x \equiv \pm\cos^2 x \pm \sin^2 x$ or equivalent e.g. $\cos 2x \equiv \pm 1 \pm 2\sin^2 x$ or $\cos 2x \equiv \pm 2\cos^2 x \pm 1$ to obtain an expression in terms of $\sin x$ and $\cos x$ only.

dM1: Expands and collects terms to obtain 2 terms in the numerator and 2 terms in the denominator. There must have been no errors in their identities for $\sin 2x$ or $\cos 2x$

A1*: Reaches the right hand side with sufficient working shown.
For this mark condone minor notational errors (e.g. a missing x) but not consistent poor notation throughout such as mixed variables.

(b)	$\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=2\sec^2 3\theta-8\Rightarrow\tan 3\theta=2\sec^2 3\theta-8$ $\Rightarrow\tan 3\theta=2\left(1+\tan^2 3\theta\right)-8$	M1
	$\Rightarrow 2\tan^2 3\theta-\tan 3\theta-6=0$ or e.g. $\Rightarrow 2\tan^2 3\theta-\tan 3\theta=6$	A1
	$\left(2\tan 3\theta+3\right)\left(\tan 3\theta-2\right)=0\Rightarrow\tan 3\theta=-\dfrac{3}{2},\ 2$ $\tan 3\theta=-\dfrac{3}{2}\Rightarrow 3\theta=...\Rightarrow\theta=...$ or $\tan 3\theta=2\Rightarrow 3\theta=...\Rightarrow\theta=...$	dM1
	$\theta=0.37,\ 0.72,\ 1.42$	A1A1
		(5)
		Total 8

Allow if a different variable used (such as *x*).
For mixed variables allow the M's but only allow the first A
(and final A's) if recovered or interpreted correctly as the same variable.

M1: Writes the lhs as $k\tan 3\theta$ and applies $\sec^2 3\theta=\pm1\pm\tan^2 3\theta$ on the rhs.
A1: Reaches the correct 3TQ in $\tan 3\theta$ with terms collected but not necessarily all on one side.
dM1: Solves their 3TQ in $\tan 3\theta$ by any method (usual rules) including a calculator (you may need to check) to obtain at least one value for $\tan 3\theta$ and uses correct processing to obtain at least one value for θ e.g. finds inverse tan and divides by 3.
A1: Awrt at least one of these answers. Must be in radians and must come from the correct 3TQ.
A1: Awrt these answers and no extras in range. Must be in radians.

Note that there may be more convoluted methods which include writing the lhs as $k\dfrac{\sin 3\theta}{\cos 3\theta}$ e.g.

$$\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}=2\sec^2 3\theta-8\Rightarrow\frac{\sin 3\theta}{\cos 3\theta}=2\sec^2 3\theta-8=\frac{2}{\cos^2 3\theta}-8$$

$$\Rightarrow\sin 3\theta\cos 3\theta=2-8\cos^2 3\theta\Rightarrow\frac{1}{2}\sin 6\theta=2-8\left(\frac{\cos 6\theta+1}{2}\right)$$

$$\Rightarrow\sin 6\theta+8\cos 6\theta=-4$$

$$\Rightarrow\sqrt{65}\sin\left(6\theta+1.4464\right)=-4\Rightarrow\sin\left(6\theta+1.4464\right)=-\frac{4}{\sqrt{65}}\Rightarrow\theta=...$$

$$\theta=0.37,\ 0.72,\ 1.42$$

In such cases, the first M mark will only be scored when they reach an equation they can solve directly e.g. in the above example when they reach $\sqrt{65}\sin\left(6\theta+1.4464\right)=-4$

If you are in doubt whether an approach deserves any credit use Review.