Question Number	Scheme	Marks
1(a)	$\text{gf}(1) = \text{g}\left(\frac{2(1)}{3\times1+1}\right) = 4-\left(\frac{2(1)}{3\times1+1}\right)^2$	M1
	$=\frac{15}{4}$ oe	A1
		(2)
(b)	$\text{f}(x) \ldots 0$ or $\text{f}(x) < \frac{2}{3}$	B1
	$0 \text{ ,, } \text{f}(x) < \frac{2}{3}$	B1
		(2)

Notes:

(a)

M1: Correct order of operations so requires an attempt to find f (1) and then apply g to their f (1)

Allow for an attempt to substitute x = 1 into $4-\left(\frac{2x}{3x+1}\right)^2$

A1: Correct value. Allow equivalents e.g. 3.75, $3\frac{3}{4}$

Correct answer only scores both marks.

(b) Allow the use of y or f but **not** x in place of f (x) for both marks.

B1: One correct bound e.g. either $\text{f}(x) \ldots 0$ or $\text{f}(x) < \frac{2}{3}$ which may be seen embedded in another inequality.

Allow **exact** equivalents for $\frac{2}{3}$ including $0.\dot{6}$

B1: Correct range. Allow equivalent notation e.g. $0 \text{ ,, } \text{f} < \frac{2}{3}$, $y \in \left[0, \frac{2}{3}\right)$, $\left[0, \frac{2}{3}\right)$, $\frac{2}{3} > y \ldots 0$

Allow written as separate inequalities e.g. $\text{f}(x) \ldots 0$, $\text{f}(x) < \frac{2}{3}$, $\text{f}(x) \ldots 0$ and $\text{f}(x) < \frac{2}{3}$

but **not** e.g. $\text{f}(x) \ldots 0$ or $\text{f}(x) < \frac{2}{3}$

Allow y or f for f (x).

Allow **exact** equivalents for $\frac{2}{3}$ including $0.\dot{6}$

(c)	$y=\frac{2x}{3x+1}\Rightarrow 3xy+y=2x\Rightarrow 3xy-2x=-y\Rightarrow x(3y-2)=-y$ or $x=\frac{2y}{3y+1}\Rightarrow 3xy+x=2y\Rightarrow 3xy-2y=-x\Rightarrow y(3x-2)=-x$	M1
	$\Rightarrow \mathrm{f}^{-1}(x)=\frac{x}{2-3x}$	A1
		(2)
(d)	$\mathrm{f}^{-1}(x)=\mathrm{f}(x)$ **or** $\mathrm{f}^{-1}(x)=x$ **or** $\mathrm{f}(x)=x$ $"\frac{x}{2-3x}"=\frac{2x}{3x+1}$ **or** $"\frac{x}{2-3x}"=x$ **or** $\frac{2x}{3x+1}=x$ $\Rightarrow x(3x+1)=2x(2-3x)$ **or** $x=x(2-3x)$ **or** $2x=x(3x+1)$ $\Rightarrow x=...$	M1
	$x=0,\ \frac{1}{3}$	A1
		(2)
		Total 8

(c)

M1: Attempts to change the subject.
If making x the subject, they must proceed as far as getting the two x terms on one side of the equation and the term not in x on the other and factorise out the x.
If making y the subject, they must swap x and y first and then proceed as far as getting the two y terms on one side of the equation and the term not in y on the other and factorise out the y.

A1: Correct expression $\left(\mathrm{f}^{-1}(x)=\right)\frac{x}{2-3x}$ oe e.g. $\left(\mathrm{f}^{-1}(x)=\right)\frac{-x}{3x-2}$
Accept equivalent expressions and isw after a correct answer is seen.
Do **not** be concerned with the lhs and just look for the correct expression in terms of x.
You can also ignore any reference to a domain as this is **not** required.

(d)

M1: Sets up one of the required equations e.g.

$$\text{their } \mathrm{f}^{-1}(x)=\mathrm{f}(x) \text{ **or** their } \mathrm{f}^{-1}(x)=x \text{ **or** } \mathrm{f}(x)=x$$

and proceeds to obtain at least one value for x.
If using their $\mathrm{f}^{-1}(x)$ it must be a function of x.
Condone copying slips as long as the intention is clear.

A1: Correct values $x=0,\ \frac{1}{3}$. Allow exact equivalents for $\frac{1}{3}$ including $0.\dot{3}$
Do not isw e.g. if they clearly reject $x = 0$ then score A0
Ignore any attempts to find the y values.