Question Number	Scheme	Marks
8 (i)	States or uses $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$	B1
	Uses both $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	$3\text{cosec}\,\theta = 8\cos\theta \Rightarrow \sin 2\theta = \dfrac{3}{4}$	A1
	$\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{3}{4}\right) = \text{awrt } 0.424, \text{awrt } 1.15$	M1 A1
		(5)
(ii)	$\dfrac{\tan 2x - \tan 70°}{1+\tan 2x\tan 70°} = -\dfrac{3}{8} \Rightarrow \tan(2x-70°) = -\dfrac{3}{8}$	M1 A1
	Correct order of operations $x = \dfrac{\arctan\left(-\dfrac{3}{8}\right)+70°}{2}$	dM1
	awrt $24.7°$, awrt $114.7°$	A1
		(4)
		(9 marks)

(i)

B1: States or uses $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$

M1: Uses both $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to set up an equation in $\sin 2\theta$

A1: $\sin 2\theta = \dfrac{3}{4}$

M1: Correct method of solving an equation of the form $\sin 2\theta = k$, $|k|<1$ to find at least one value for θ. May be solving in degrees for this mark $(24.30°, 65.70°)$

A1: $\theta = \text{awrt } 0.424, \text{awrt } 1.15$ and no other values in the range (isw for incorrect rounding).

(ii) Main scheme

M1: Uses the compound angle identities to reach $\tan(2x \pm 70°) = \pm\dfrac{3}{8}$

A1: $\tan(2x-70°) = -\dfrac{3}{8}$

dM1: Correct order of operations to find a value for *x*. Condone if radians mode used, e.g.

$2x - 70 = -0.3587... \Rightarrow x = \dfrac{70 - 0.3587...}{2} = 34.82...$ May be scored from

$2x + 70 = ... \rightarrow x = \dfrac{-70 - ....}{2}$ Condone errors in finding the second solution such at $180 - PV$.

A1: Both awrt $24.7°$, awrt $114.7°$ and no other values in the range (isw for incorrect rounding)

(ii) Alt method 1

M1: Cross multiplies and makes $\tan 2x$ the subject. Condone $\tan 70°$ being replaced by 2.75

A1: $\tan 2x = \dfrac{8\tan 70° - 3}{8 + 3\tan 70°} = \text{awrt } 1.17$

dM1: Correct order of operations to find one value for *x*. See note on main scheme.
Implied by a correct value for their $\tan 2x = \text{awrt } 1.17$ provided the previous M has been scored

A1: Both awrt $24.7°$, awrt $114.7°$ and no other values in the range (isw for incorrect rounding)

(ii) Alt method 2

M1: Applies double angle formula and multiplies through to achieve a quadratic in tan x (condoning slips and not necessarily gathered)

$$\frac{\frac{2\tan x}{1-\tan^2 x}-\tan 70°}{1+\frac{2\tan x}{1-\tan^2 x}\tan 70°}=-\frac{3}{8}\Rightarrow 16\tan x-8\tan 70°\left(1-\tan^2 x\right)=-3\left(1-\tan^2 x+2\tan x\tan 70°\right)$$

$$\left(\Rightarrow t^2\left(8\tan 70°-3\right)+t\left(16+6\tan 70°\right)+3-8\tan 70°=0\ \left(18.98t^2+32.48t-18.98=0\right)\right)$$

A1: A correct value for tan x, tan x = awrt 0.4604 or awrt −2.172

dM1: Applies arctan to achieve at least one value for x.

A1: As main scheme.

Alt (i)	States or uses $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ (or equivalent identity)	B1
	$3\operatorname{cosec}\theta=8\cos\theta\Rightarrow 9\operatorname{cosec}^2\theta=64\cos^2\theta$ and uses both $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\sin^2\theta+\cos^2\theta=1$ to get equation in one trig term	M1
	$\Rightarrow 9=64\sin^2\theta\left(1-\sin^2\theta\right)\Rightarrow\sin^2\theta=\frac{4\pm\sqrt{7}}{8}$	A1
	$\Rightarrow\theta=\arcsin\left(\sqrt{\frac{4\pm\sqrt{7}}{8}}\right)=$ awrt 0.424, awrt 1.15	M1 A1
		(5)

B1: States or uses $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ or in variations a similar correct reciprocal identity (e.g. $\cot\theta=\frac{\cos\theta}{\sin\theta}$) during the proof.

M1: Squares both sides and uses both reciprocal identity and Pythagorean identity to set up an equation in one trig term only.

A1: $\sin^2\theta=\frac{4\pm\sqrt{7}}{8}$ note it is the same roots for $\cos^2\theta$

M1: Correct method of solving an equation of the form $\sin^2\theta=k$ or $\cos^2\theta=k$, $0<k<1$ to find at least one value for θ. May be solving in degrees for this mark $(24.30°, 65.70°)$

A1: $\theta=$ awrt 0.424, awrt 1.15 and no other values in the range. Under this method the extra solutions formed via squaring will need to be rejected to score this mark.

Question Number	Scheme	Marks
9. (a)	$\frac{1}{4}$	B1 **(1)**
(b)	$f'(x)=\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}$	M1 A1
	Sets $\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}=0\Rightarrow\ln(4x)=-2$	dM1
	$Q\left(\frac{1}{4e^2},-\frac{6}{e}\right)$	A1 A1 **(5)**
(c)	Attempts $-2\times y$ co-ordinate of Q	M1
	Range $g(x)\geqslant\frac{12}{e}$	A1ft **(2)**
		(8 marks)

(a)

B1: $\frac{1}{4}$ or exact equivalent. May be seen as the x coordinate in a coordinate pair.

(b)

M1: Attempts to differentiate using the product rule and achieves $f'(x)=\frac{a}{\sqrt{x}}\ln(4x)+b\sqrt{x}\times\frac{1}{x}$

A1: $f'(x)=\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}$. Note the $\frac{1}{x}$ may be seen as $\frac{4}{4x}$

dM1: Sets $f'(x)=0$ and proceeds to $\ln(4x)=k$. The "=0" may be implied by the attempt to solve. Condone attempts where multiplying through by $\sqrt{x}$ occurs before setting equal to 0.

A1: $x=\frac{1}{4e^2}$ oe but allow awrt 0.034 following correct equation.

A1: $Q\left(\frac{1}{4e^2},-\frac{6}{e}\right)$ Both coordinates correct and simplified, but may be listed separately, x=…, y = … and allow if y is found in (c). Accept negative powers of e, but must be simplified coefficients and simplified powers.

(c)

M1: Attempts $-2\times$ their y co-ordinate of Q. Also allow following decimal answer

A1ft: Range $g(x)\geqslant\frac{12}{e}$ following through on their negative y value for Q. Accept with y instead of g(x) and wtih negative powers or expression in e following their (b), e.g. $y\geqslant 12e^{-1}$ Accept decimals following a decimal answer to (b). Do not accept f(x).