Question Number	Scheme	Marks
8 (a)	52 b.p.m.	B1
		(1)
(b)	32 b.p.m.	B1
		(1)
(c)	$\dfrac{\mathrm{d}H}{\mathrm{d}t} = -8\,\mathrm{e}^{-0.2t} + 18\,\mathrm{e}^{-0.9t}$	M1, A1
	Sets $-8\,\mathrm{e}^{-0.2t} + 18\,\mathrm{e}^{-0.9t} = 0 \Rightarrow 4\mathrm{e}^{0.7t} = 9$	dM1
	$\Rightarrow 0.7t = \ln\dfrac{9}{4} \Rightarrow t = ...$	M1
	T = 1.158 (minutes)	A1
		(5)
(d)	$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t} \Rightarrow \mathrm{e}^{-0.2t} = \dfrac{1+4\,\mathrm{e}^{-0.9t}}{8}$	M1
	$\Rightarrow \mathrm{e}^{0.2t} = \dfrac{8}{1+4\,\mathrm{e}^{-0.9t}} \Rightarrow t = 5\ln\left(\dfrac{8}{1+4\,\mathrm{e}^{-0.9t}}\right)$	A1*
		(2)
(e)	$t_2 = 5\ln\left(\dfrac{8}{1+4\mathrm{e}^{-0.9\times 10}}\right) = ...$	M1
	$\left(t_2\right)$ = awrt 10.3947	A1
	(M)= 10.3955	A 1
		(3)
		Total 12

(a)

B1: 52 b.p.m. Units are not required. Check for answer given in the body of the question

(b)

B1: 32 b.p.m. . Units are not required. Check for answer given in the body of the question

(c)

M1: Achieves $\dfrac{\mathrm{d}H}{\mathrm{d}t} = P\,\mathrm{e}^{-0.2t} + Q\mathrm{e}^{-0.9t}$ where P and Q are constants

A1: $\dfrac{\mathrm{d}H}{\mathrm{d}t} = -8\,\mathrm{e}^{-0.2t} + 18\,\mathrm{e}^{-0.9t}$ but allow un-simplified versions

dM1: Sets $P\,\mathrm{e}^{-0.2t} + Q\,\mathrm{e}^{-0.9t} = 0 \Rightarrow m\mathrm{e}^{0.7t} = n$ or $m\mathrm{e}^{-0.7t} = n$ where $m \times n > 0$

M1: Solves an equation of the form $m\mathrm{e}^{\pm kt} = n$ where $m \times n > 0$ using correct ln work.
It must proceed from a correct form of the derivative (so the first M1).

Look for $m\mathrm{e}^{\pm kt} = n \Rightarrow \mathrm{e}^{\pm kt} = \dfrac{n}{m} \Rightarrow \pm kt = \ln\left(\dfrac{n}{m}\right) \Rightarrow t = ...$

A1: T = awrt 1.158 minutes following the award of all previous marks. You can ignore units

……

An alternative for the last three marks in part (c) are;

dM1: $P\mathrm{e}^{-0.9t} = Q\mathrm{e}^{-0.2t} \Rightarrow \ln P - 0.9t = \ln Q - 0.2t$

M1: Collects terms in t and proceeds to t =… Under this method it is dependent upon both previous M's

A1: T = awrt 1.158 minutes following the award of all previous marks. Units are not required

……
There may be answers produced from a calculator. In cases such as this we are going to apply the rule 'answers without full working may not gain credit.'

$\dfrac{\mathrm{d}H}{\mathrm{d}t} = -8\,\mathrm{e}^{-0.2t} + 18\,\mathrm{e}^{-0.9t} = 0 \Rightarrow t = 1.158$ scores 1,1,0,0,0

…….

(d)

M1: Sets $H = 37$ and proceeds to make $\mathrm{e}^{\pm 0.2t}$ the subject

A1*: Proceeds to the given answer showing all necessary steps.

Note that this is a given answer so the steps must be logical without huge jumps

Acceptable proofs:

Example 1

$$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t} \Rightarrow \mathrm{e}^{-0.2t} = \frac{1 + 4\,\mathrm{e}^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1 + 4\,\mathrm{e}^{-0.9t}}{8}\right)$$

$$\Rightarrow t = -5\ln\left(\frac{1 + 4\,\mathrm{e}^{-0.9t}}{8}\right)$$

$$\Rightarrow t = 5\ln\left(\frac{8}{1 + 4\,\mathrm{e}^{-0.9t}}\right)$$

Example 2

$$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t} \Rightarrow \mathrm{e}^{-0.2t} = \frac{1 + 4\,\mathrm{e}^{-0.9t}}{8} \Rightarrow \mathrm{e}^{0.2t} = \frac{8}{1 + 4\,\mathrm{e}^{-0.9t}} \Rightarrow 0.2t = \ln\left(\frac{8}{1 + 4\,\mathrm{e}^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1 + 4\,\mathrm{e}^{-0.9t}}\right)$$

Unacceptable proof:

$$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t} \Rightarrow \mathrm{e}^{-0.2t} = \frac{1 + 4\,\mathrm{e}^{-0.9t}}{8} \Rightarrow -0.2t = \ln\left(\frac{1 + 4\,\mathrm{e}^{-0.9t}}{8}\right)$$

$$\Rightarrow t = 5\ln\left(\frac{8}{1 + 4\,\mathrm{e}^{-0.9t}}\right)$$

They need to deal with 0.2 and $-$sign in separate steps

……

Alt (d) lns can be taken earlier. The M1 is scored for making 0.2 *t* the subject. See below

$$37 = 32 + 40\,\mathrm{e}^{-0.2t} - 20\,\mathrm{e}^{-0.9t}$$

$$\Rightarrow 40\mathrm{e}^{-0.2t} = 5 + 20\,\mathrm{e}^{-0.9t} \Rightarrow \ln 40 - 0.2t = \ln\left(5 + 20\,\mathrm{e}^{-0.9t}\right) \Rightarrow 0.2t = \ln 40 - \ln\left(5 + 20\,\mathrm{e}^{-0.9t}\right)$$

We would need to see three further steps. E.g.

$$0.2t = \ln\left(\frac{40}{5 + 20\,\mathrm{e}^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{40}{5 + 20\,\mathrm{e}^{-0.9t}}\right) \Rightarrow t = 5\ln\left(\frac{8}{1 + 4\,\mathrm{e}^{-0.9t}}\right)$$

……
……..

(e)

M1: Uses the iteration formula once. Allow for the embedded 10 leading to a value or sight of t_2 = awrt 10.4

A1: $\left(t_2\right)$ = awrt 10.3947

A1: $(M) = 10.3955$ following some evidence of previous iterations, e.g. sight of

$t_2 = 5\ln\left(\dfrac{8}{1+4\mathrm{e}^{-0.9\times10}}\right) = $ awrt 10.4.... This is NOT awrt so just 10.3955 here

Question Number	Scheme	Marks
9 (a)	$\mathrm{f}'(x) = \dfrac{(2x+1)\times(12x+4)-2\left(6x^2+4x-2\right)}{(2x+1)^2}$	M1 A1
	$= \dfrac{12x^2+12x+8}{(2x+1)^2}$ o.e	A1
		(3)
(b)	At $x=2 \Rightarrow \mathrm{f}'(x) = \dfrac{12\times2^2+12\times2+8}{(2\times2+1)^2} = \left(\dfrac{16}{5}\right)$	M1
	Full method of normal $y-6 = -\dfrac{5}{16}(x-2)$	dM1
	$16y-96 = -5x+10 \Rightarrow \ 16y+5x = 106$ *	A1*
		(3)
(c)	Any correct value $A=3,\ B=\dfrac{1}{2}$ *or* $D=-\dfrac{5}{2}$	B1
	$6x^2+4x-2 = (Ax+B)(2x+1)+D \Rightarrow$ Values of A, B and D	M1
	$3x+\dfrac{1}{2}+\dfrac{-5/2}{2x+1}$ o.e	A1
		(3)
(d)	$\displaystyle\int 3x+\frac{1}{2}+\frac{-5/2}{2x+1}\,\mathrm{d}x = \frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)$	M1, A1 ft
	Area under curve = $\left[\frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)\right]_{\frac{1}{3}}^{2} = \left(\frac{3}{2}\times2^2+\frac{1}{2}\times2-\frac{5}{4}\ln(5)\right)-\left(\frac{3}{2}\times\left(\frac{1}{3}\right)^2+\frac{1}{2}\times\left(\frac{1}{3}\right)-\frac{5}{4}\ln\left(\frac{5}{3}\right)\right)$	dM1
	Area of R = $\displaystyle\int_{\frac{1}{3}}^{2} 3x+\frac{1}{2}+\frac{-5/2}{2x+1}\mathrm{d}x+\frac{1}{2}\times6\times\left(\frac{106}{5}-2\right)$	M1
	Area of $R = \dfrac{964}{15}-\dfrac{5}{4}\ln3$	A1
		(5)
		Total 14

(a)

M1: Condoning slips, this is for an attempt using

- the quotient rule to obtain $\mathrm{f}'(x) = \dfrac{(2x+1)\times(Ax+B)-C\left(6x^2+4x-2\right)}{(2x+1)^2}$ Must be '–' on the numerator
- the chain rule on $3x+A+\dfrac{B}{2x+1} \Rightarrow \mathrm{f}'(x) = 3\pm D(2x+1)^{-2}$
- the product rule on $\left(6x^2+4x-2\right)(2x+1)^{-1} \Rightarrow \mathrm{f}'(x) = (2x+1)^{-1}\times(Ax+B)-C\left(6x^2+4x-2\right)(2x+1)^{-2}$ where A, B and C are > 0