Question Number	Scheme	Marks
8	$h = 1.5x - 0.5x\,\mathrm{e}^{0.02x}$	
(a)	$0 = 1.5d - 0.5d\,\mathrm{e}^{0.02d} \Rightarrow \mathrm{e}^{0.02d} = 3$	M1
	$\Rightarrow 0.02d = \ln 3 \Rightarrow d = 54.93$	dM1 A1
		(3)
(b)	$\left(\frac{\mathrm{d}h}{\mathrm{d}x} =\right) 1.5 - \left(0.5\mathrm{e}^{0.02x} + 0.5x \times 0.02\,\mathrm{e}^{0.02x}\right)$	M1A1
	Sets $0 = 1.5 - 0.5\mathrm{e}^{0.02x} - 0.5x \times 0.02\,\mathrm{e}^{0.02x} \Rightarrow \mathrm{e}^{0.02x}\left(0.5 + 0.5x \times 0.02\right) = 1.5$	
	$\Rightarrow \mathrm{e}^{0.02x} = \frac{1.5}{\left(0.5 + 0.5x \times 0.02\right)}$	dM1
	$\Rightarrow \mathrm{e}^{0.02x} = \frac{1.5}{\left(0.5 + 0.5x \times 0.02\right)} = \frac{150}{50 + x} \Rightarrow x = 50\ln\left(\frac{150}{50 + x}\right)$ *	A1*
		(4)
(c)	(i) awrt 31.43	M1 A1
	(ii) awrt 30.88 metres **(including units)**	A1
		(3)
		(10 marks)

(a) **Allow to be solved in either *x* or *d***

M1: Sets $h = 0$, cancels by d (or takes out a factor of d) and proceeds to a form $a\,\mathrm{e}^{\pm 0.02d} = b$ where $ab > 0$ and a may be 1. Condone a misread or miscopy proceeding to the form $a\,\mathrm{e}^{\pm 0.2d} = b$

dM1: Solves the equation of the required form using logs. It is dependent on the previous method mark. Look for an expression for d (or x) or a multiple of d (or x) before proceeding to a value for d.

e.g. $\mathrm{e}^{0.02d} = 3 \Rightarrow d = \frac{\ln 3}{0.02}$

e.g. $\mathrm{e}^{-0.02d} = \frac{1}{3} \Rightarrow -0.02d \ln \mathrm{e} = \ln \frac{1}{3} \Rightarrow d =$

Condone the slip on the index to be $\pm 0.2d$

It cannot be implied by a numerical value for *d*.

A1: 54.93 (m) cao (units not required but if given they must be correct) following correct log work seen and at least one intermediate stage of working. Condone $x = 54.93$. isw if they round after a correct answer is seen.

Minimum acceptable e.g. $\mathrm{e}^{0.02d} = 3 \Rightarrow d = \frac{\ln 3}{0.02} = 54.93$ (M1dM1A1)

e.g. $0 = 1.5d - 0.5d\,\mathrm{e}^{0.02d} \Rightarrow 0.02d = \ln 3 \Rightarrow d = 54.93$ (M1dM1A1)

e.g. $0 = 1.5d - 0.5d\,\mathrm{e}^{0.02d} \Rightarrow 3 = \mathrm{e}^{0.02d} \Rightarrow d = 54.93$ (M1dM0A0)

(b) **If no attempt is seen for (b) then allow differentiation seen in (a) to score in (b).**

M1: Attempts the product rule on $x\mathrm{e}^{0.02x}$ achieving $\pm Ax\mathrm{e}^{0.02x} \pm B\mathrm{e}^{0.02x}$ where A can be 1. It is likely to be part of an expression.

Condone a misread or miscopy proceeding to the form $\pm Ax\mathrm{e}^{0.2x} \pm B\mathrm{e}^{0.2x}$

A1: $\left(\frac{\mathrm{d}h}{\mathrm{d}x} =\right) 1.5-\left(0.5\mathrm{e}^{0.02x}+0.5x\times 0.02\,\mathrm{e}^{0.02x}\right)$ o.e.

dM1: Sets $0=1.5-"0.5\mathrm{e}^{0.02x}-0.5x\times 0.02\,\mathrm{e}^{0.02x}"$, attempts to make $\mathrm{e}^{\pm 0.02x}$ (or $C\mathrm{e}^{\pm 0.02x}$) the subject and proceeds to the form $C\mathrm{e}^{\pm 0.02x}=\frac{D}{E+Fx}$ where C can be 1.

May see $0=150-"50\mathrm{e}^{0.02x}-x\mathrm{e}^{0.02x}" \Rightarrow \mathrm{e}^{0.02x}=\frac{150}{50+x}$

Condone slips in their rearrangement but they must take out a factor of $\mathrm{e}^{\pm 0.02x}$ (or $C\mathrm{e}^{\pm 0.02x}$) and divide by their bracket. If they take logs of both sides first, the rearrangement must be correct. Condone invisible brackets to be recovered or implied by further work.

Condone a misread or miscopy proceeding from $0=1.5-"0.5\mathrm{e}^{0.02x}-0.5x\times 0.02\,\mathrm{e}^{0.02x}"$ and attempting to make $\mathrm{e}^{\pm 0.2x}$ (or $C\mathrm{e}^{\pm 0.2x}$) the subject.

A1*: Achieves the given answer with no errors seen provided all previous marks have been scored. Do not allow this mark to be scored for proceeding directly from $\mathrm{e}^{0.02x}\left(0.5+0.5x\times 0.02\right)=1.5$ to $x=50\ln\left(\frac{150}{50+x}\right)$.

We must see either $\mathrm{e}^{0.02x}=\frac{1.5}{\left(0.5+0.5x\times 0.02\right)}$ o.e. or an unsimplified expression for x

e.g. $x=50\ln\left(\frac{1.5}{0.5+0.01x}\right)$ before achieving the given answer.

(c)(i) **Check by the question. If there is a contradiction between answers, the answer in the main body of the work takes precedence.**

M1: Attempts the iteration formula "correctly" seen once.

E.g. Award for $x_2=50\ln\left(\frac{150}{80}\right)$ or awrt 31.4

A1: awrt 31.43

(c)(ii)

A1: awrt 30.88 m **(including units)**

Question Number	Scheme	Marks
9 (a)	$(k=)\ 4\sin^2\left(\frac{\pi}{3}\right)-1=2$ *	B1*
		(1)
(b)	(i) $x=4\sin^2 y-1\Rightarrow \quad \frac{\mathrm{d}x}{\mathrm{d}y}=8\sin y\cos y$ o.e.	M1 A1
	(ii) Attempts either $\sin^2 y=\frac{\pm x\pm 1}{4}$ or $\cos^2 y=\pm 1\pm\frac{x+1}{4}$ (both for dM1)	M1 dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sin y\cos y}=\frac{1}{8\times\sqrt{\frac{x+1}{4}}\times\sqrt{\pm 1\pm\frac{x+1}{4}}}=\frac{1}{2\sqrt{x+1}\sqrt{3-x}}$ *	ddM1 A1*
		(6)
(c)	At $x=2$, gradient of curve = $\frac{1}{2\sqrt{3}}\Rightarrow$ gradient of normal is $-2\sqrt{3}$	M1
	Point *N* (base length of triangle) is solution of	
	$y-\frac{\pi}{3}=-2\sqrt{3}\left(x-2\right)\Rightarrow x=2+\frac{\pi}{6\sqrt{3}}$ (= 2.3...)	dM1
	Area = $\frac{1}{2}\times\left(2+\frac{\pi}{6\sqrt{3}}\right)\times\frac{\pi}{3}=\frac{\pi}{3}+\frac{\pi^2}{36\sqrt{3}}$	A1
		(3)
		(10 marks)

(a)

B1*: Verifies that $k=2$ with no errors seen. (Condone $x=2$) Do not allow this mark if rounded numbers are seen in their solution for e.g. $\frac{\pi}{3}$. They may work in degrees and substitute in $60°$ which is acceptable. Look for as a minimum e.g. $(k=)\ 4\sin^2\frac{\pi}{3}-1=2$ Do not allow $k=4\sin\left(\frac{\pi}{3}\right)^2-1=2$

If they rearrange the equation to e.g. $y=\sin^{-1}\left(\sqrt{\frac{x+1}{4}}\right)$ and substitute in $x=2$ proceeding to e.g. $\frac{\pi}{3}$ then they must conclude that $k=2$ (or a preamble followed by a minimal conclusion e.g. QED, tick)

(b) (i)

M1: Differentiates to a form $p\sin y\cos y$ or equivalent. Indices do not need to be processed for this mark. e.g. ...$\sin^{2-1} y\cos y$

May use the identity $\cos 2y=\pm 1\pm 2\sin^2 y\Rightarrow\ x=4\sin^2 y-1=1-2\cos 2y\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=q\sin 2y$

A1: Correct lhs and rhs. For $\frac{\mathrm{d}x}{\mathrm{d}y}=8\sin y\cos y$ or $4\sin 2y$. Note that $\frac{\mathrm{d}y}{\mathrm{d}x}=...$ is A0. isw for this mark after a correct answer is seen. You may see $\frac{\mathrm{d}x}{\mathrm{d}y}$ (or allow x') on an earlier line which is fine for this mark. May be seen at the start of (b)(ii). Indices must be processed for this mark.