Question Number	Scheme	Marks
7 (a)	$\sqrt{2}\sin(x+45°)=\cos(x-60°)$	
	$\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)=\cos x\cos 60°+\sin x\sin 60°$	M1 A1
	$\sin x+\cos x=\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x$	
	$\cos x=(\sqrt{3}-2)\sin x$	M1
	$\tan x\left(=\frac{1}{\sqrt{3}-2}=\frac{\sqrt{3}+2}{-1}\right)=-2-\sqrt{3}$ *	A1*
		(4)
(b)	States or uses $x+45°=2\theta$ o.e.	B1
	Proceeds from e.g. $\tan(2\theta-45°)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°,285°$	M1
	Correct order of operations to find one angle	dM1
	$\theta=75°,165°$	A1
		(4)
		(8 marks)

(a) Condone working in mixed variables

M1: Attempts to use the compound angle identities to produce an equation in $\sin x$ and $\cos x$

Look for $\sqrt{2}\times(\sin x\cos 45°\pm\cos x\sin 45°)=\cos x\cos 60°\pm\sin x\sin 60°$ o.e.

May be implied by a first line of e.g. $\sin x\pm\cos x=\frac{1}{2}\cos x\pm\frac{\sqrt{3}}{2}\sin x$ o.e.

Condone missing brackets.

A1: Correct equation in $\sin x$ and $\cos x$.

E.g. $\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°)=\cos x\cos 60°+\sin x\sin 60°$ o.e. (may be implied by further work). May also see use of equivalent angles e.g. $\cos(-60)°=\cos 60°$

M1: For an attempt to solve the problem. Look for an attempt (condoning slips) at two of the three elements required to complete the proof, namely

- use of correct exact trigonometric values for $\sin 45°,\cos 45°,\cos 60°,\sin 60°$
- collection of like terms in $\sin x$ and $\cos x$ or $\tan x$
- use of $\tan x=\frac{\sin x}{\cos x}$

A1*: Proceeds to the given answer with all previous marks scored.

There should be no errors in the manipulation and no bracket omissions, other than a missing trailing bracket.

e.g. $\sqrt{2}(\sin x\cos 45°+\cos x\sin 45°=\cos x\cos 60°+\sin x\sin 60°$

When proceeding from $A\sin x=B\cos x$ (where A or B may be 1) to the given answer it is acceptable to proceed from $\frac{1}{2}\cos x=\frac{-2+\sqrt{3}}{2}\sin x$ to the given answer as evidence of the use of $\tan x=\frac{\sin x}{\cos x}$.

Condone poor notation to be recovered provided the final answer line is written correctly. Condone working in mixed variables provided the final given answer is all in terms of x.

Allow surd work to be done via a calculator.

(b)

B1: States or uses $x+45°=2\theta$ o.e. e.g. $\theta=\dfrac{x+45°}{2}$

This is implied for sight of the equation $\tan\left(2\theta-45°\right)=-2-\sqrt{3}$

M1: Proceeds from $\tan\left(2\theta\pm\alpha°\right)=-2-\sqrt{3}\Rightarrow 2\theta\pm\alpha°=105°$ or $285°$ where $\alpha\neq 0$

The attempt must either achieve an angle of $105°$ or $285°$ or equivalent expression (Allow in radians (3sf) which are 1.83, 4.97), or allow a general solution of e.g. $2\theta\pm\alpha°=-75°+\left(180n\right)°$

Maybe implied by further work which is not $75°$ or $165°$

dM1: Correct order of operations to solve their $2\theta\pm\alpha°=...$

This is dependent on the previous method mark.

Note that $\tan\left(2\theta-45°\right)=-2-\sqrt{3}\Rightarrow 75°$ does not imply this mark. We must see either e.g. $2\theta-45°=105°\Rightarrow\theta=...$ or some intermediate stage before seeing $75°$

A1: Both angles $\theta=75°,165°$ with no others given within the range

Note that $\tan\left(2\theta-45°\right)=-2-\sqrt{3}\Rightarrow 2\theta-45°=105°\Rightarrow\theta=75°,165°$ is acceptable for full marks

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Alt (b) via use of $\cos(2\theta-105°)=\cos 2\theta\cos 105°+\sin 2\theta\sin 105°$

$\sqrt{2}\sin 2\theta=\cos(2\theta-105°)\Rightarrow\tan 2\theta=\dfrac{\cos 105°}{\sqrt{2}-\sin 105°}\Rightarrow\theta=75°,165°$

Note the order of the marks needs to match up to the main scheme so 0110 is possible.

B1: For achieving $\tan 2\theta=-\dfrac{\sqrt{3}}{3}$ o.e. so allow $\tan 2\theta=\dfrac{\cos 105°}{\sqrt{2}-\sin 105°}=$ awrt -0.58

Or via double angle identities $\sqrt{3}\tan^2\theta-6\tan\theta-\sqrt{3}=0$ o.e.

M1: Attempts to use the compound angle identities (allowing sign slips when using them) to reach a form $\tan 2\theta=k$ where k is a constant not $-2-\sqrt{3}$ (or expression in trigonometric terms such as $\cos 105°$ as seen above). Allow $2\theta=-30°$ o.e. (allow in radians $-\dfrac{\pi}{6}$) to imply this mark. Do not be concerned by the mechanics of their rearrangement.

Alternatively, via double angle identities reaches a 3TQ in $\tan\theta$

dM1: Correct order of operations from $\tan 2\theta=k$ proceeding to $\theta=...$ e.g. $2\theta=-30°\Rightarrow\theta=-15°$ **(which must be in degrees)** can score this mark. You may need to check this if $\theta=\dfrac{\tan^{-1}k}{2}=...$ is not written

Alternatively, correctly solves their $\sqrt{3}\tan^2\theta-6\tan\theta-\sqrt{3}=0$ proceeding to $\theta=...$

A1: Both angles $\theta=75°,165°$ with no others given within the range

Question Number	Scheme	Marks
8	$h = 1.5x - 0.5x\,\mathrm{e}^{0.02x}$	
(a)	$0 = 1.5d - 0.5d\,\mathrm{e}^{0.02d} \Rightarrow \mathrm{e}^{0.02d} = 3$	M1
	$\Rightarrow 0.02d = \ln 3 \Rightarrow d = 54.93$	dM1 A1
		(3)
(b)	$\left(\frac{\mathrm{d}h}{\mathrm{d}x} =\right) 1.5 - \left(0.5\mathrm{e}^{0.02x} + 0.5x \times 0.02\,\mathrm{e}^{0.02x}\right)$	M1A1
	Sets $0 = 1.5 - 0.5\mathrm{e}^{0.02x} - 0.5x \times 0.02\,\mathrm{e}^{0.02x} \Rightarrow \mathrm{e}^{0.02x}\left(0.5 + 0.5x \times 0.02\right) = 1.5$	
	$\Rightarrow \mathrm{e}^{0.02x} = \frac{1.5}{\left(0.5 + 0.5x \times 0.02\right)}$	dM1
	$\Rightarrow \mathrm{e}^{0.02x} = \frac{1.5}{\left(0.5 + 0.5x \times 0.02\right)} = \frac{150}{50 + x} \Rightarrow x = 50\ln\left(\frac{150}{50 + x}\right)$ *	A1*
		(4)
(c)	(i) awrt 31.43	M1 A1
	(ii) awrt 30.88 metres **(including units)**	A1
		(3)
		(10 marks)

(a) **Allow to be solved in either *x* or *d***

M1: Sets $h = 0$, cancels by d (or takes out a factor of d) and proceeds to a form $a\,\mathrm{e}^{\pm 0.02d} = b$ where $ab > 0$ and a may be 1. Condone a misread or miscopy proceeding to the form $a\,\mathrm{e}^{\pm 0.2d} = b$

dM1: Solves the equation of the required form using logs. It is dependent on the previous method mark. Look for an expression for d (or x) or a multiple of d (or x) before proceeding to a value for d.

e.g. $\mathrm{e}^{0.02d} = 3 \Rightarrow d = \frac{\ln 3}{0.02}$

e.g. $\mathrm{e}^{-0.02d} = \frac{1}{3} \Rightarrow -0.02d \ln \mathrm{e} = \ln \frac{1}{3} \Rightarrow d =$

Condone the slip on the index to be $\pm 0.2d$

It cannot be implied by a numerical value for *d*.

A1: 54.93 (m) cao (units not required but if given they must be correct) following correct log work seen and at least one intermediate stage of working. Condone $x = 54.93$. isw if they round after a correct answer is seen.

Minimum acceptable e.g. $\mathrm{e}^{0.02d} = 3 \Rightarrow d = \frac{\ln 3}{0.02} = 54.93$ (M1dM1A1)

e.g. $0 = 1.5d - 0.5d\,\mathrm{e}^{0.02d} \Rightarrow 0.02d = \ln 3 \Rightarrow d = 54.93$ (M1dM1A1)

e.g. $0 = 1.5d - 0.5d\,\mathrm{e}^{0.02d} \Rightarrow 3 = \mathrm{e}^{0.02d} \Rightarrow d = 54.93$ (M1dM0A0)