Question Number	Scheme	Marks
5 (a)	$\mathrm{f}^{-1}(22) \Rightarrow 2+5\ln x = 22 \Rightarrow \ln x = 4 \Rightarrow (x=)\ \mathrm{e}^4$	M1 A1 **(2)**
(b)	$\mathrm{g}(x) = \dfrac{6x-2}{2x+1} \Rightarrow \mathrm{g}'(x) = \dfrac{6(2x+1)-2(6x-2)}{(2x+1)^2}$	M1 A1
	States $(\mathrm{g}'(x)=)\ \dfrac{10}{(2x+1)^2} > 0$ hence increasing *	A1* **(3)**
(c)	$y = \dfrac{6x-2}{2x+1} \Rightarrow 2xy + y = 6x - 2 \Rightarrow 2xy - 6x = -y - 2$	M1
	$\Rightarrow x = \dfrac{-y-2}{2y-6}$ So $\mathrm{g}^{-1}(x) = \dfrac{-x-2}{2x-6}$ o.e.	A1
	Domain $0 < x < 3$	B1 **(3)**
(d)	Range fg is $\mathrm{fg} < 2+5\ln 3$	M1, A1 **(2)**
		(10 marks)

(a)

M1: Sets $2+5\ln x = 22$ and rearranges to $\ln x = ...$ (may be implied by further work)

Alternatively, may attempt to find $\mathrm{f}^{-1}(x)$ and substitute 22 into the expression. i.e. $\mathrm{f}^{-1}(x) = \mathrm{e}^{\frac{\pm x \pm 2}{5}}$.
Maybe implied by awrt 55.
Condone sign slips only in their rearrangement.

A1: e^4 (must be simplified and exact) isw once a correct answer is seen.

(b)

M1: Either

- attempts to use the quotient rule proceeding to the form $\dfrac{A(2x+1)-B(6x-2)}{(2x+1)^2}$ o.e. where A and B are both positive. Do not award this mark if the quotient rule is applied the wrong way round.
- attempts to use the product rule proceeding to the form $(6x-2)(2x+1)^{-1} \Rightarrow \pm C(6x-2)(2x+1)^{-2} \pm D(2x+1)^{-1}$
- writes g(x) as $3-\dfrac{5}{2x+1} = 3-5(2x+1)^{-1}$ and attempts to use the chain rule proceeding to the form $E(2x+1)^{-2}$

In all methods invisible brackets can be recovered or implied by further work.

A1: Correct unsimplified e.g. $\dfrac{6(2x+1)-2(6x-2)}{(2x+1)^2}$ or $-2(6x-2)(2x+1)^{-2}+6(2x+1)^{-1}$ or $10(2x+1)^{-2}$

A1*: Requires

- a reason as to why the expression is always positive e.g. by finding $\left(g'(x)=\right)\dfrac{10}{(2x+1)^2}>0$
- a minimal conclusion

e.g. writes $g'(x)$ as $\dfrac{10}{(2x+1)^2}$ **and deduces > 0** with minimal conclusion e.g. "it is an increasing function" or "hence increasing" **Condone use of** $\geqslant$

They may consider just the denominator separately, but they would still need to simplify the numerator to 10 and state e.g. $(2x+1)^2>0$ o.e. with a minimal conclusion

Do not accept the conclusion $g'(x)$ is an increasing function if stated instead of $g(x)$.

Note that they cannot just state e.g. "$-2(6x-2)(2x+1)^{-2}+6(2x+1)^{-1}>0$ so increasing" as it is not clear from the expression that this will be the case.

Do not withhold this mark if invisible brackets are recovered but A0* if there are errors in the method.

(c)

M1: Attempts to change the subject. Look for cross multiplication with an attempt to collect terms achieving the form $\pm 2xy \pm 6x = \pm y \pm 2$ or $\pm 2xy \pm 6y = \pm x \pm 2$. May be implied by further work.

If they start from their $y=\alpha\pm\dfrac{\beta}{2x+1}$ look for an alternative rearrangement to $\dfrac{\beta}{y\pm\alpha}=2x+1$ (or $\dfrac{\beta}{x\pm\alpha}=2y+1$)

A1: $g^{-1}(x)=\dfrac{-x-2}{2x-6}$ or equivalent such as $g^{-1}(x)=\dfrac{x+2}{6-2x}$ or may be seen as $g^{-1}(x)=\dfrac{\frac{5}{3-x}-1}{2}$ or $g^{-1}(x)=\dfrac{5}{6-2x}-0.5$

Accept $g^{-1}: x\to\dfrac{x+2}{6-2x}$ Condone $g^{-1}=...$ or $g^{-1}=y=...$ but do not allow just $y=\dfrac{x+2}{6-2x}$

B1: Correct domain $0<x<3$ o.e. e.g. $0<x\cap x<3$ or $]0,3[$ but not $0<y<3$ or $0<g^{-1}(x)<3$

Note: It is also acceptable to define g^{-1} in any variable e.g. as $g^{-1}(t)=\dfrac{t+2}{6-2t}$, $0<t<3$ as long as the variable is used consistently to score M1A1B1. If another variable is used other than x it must be fully defined e.g. $g^{-1}(t)=...$ not just $g^{-1}=...$

(d)

M1: $2+5\ln 3$ found. Allow awrt 7.5 instead of $2+5\ln 3$ for this mark only. Do not be concerned by the direction of the inequality if seen.

A1: Fully correct. $\left(-\infty<\right)\text{fg}<2+5\ln 3$ o.e. $5\ln 3$ must be exact. Do not be too concerned by alternative notation to fg as long as it is not x. There are many alternatives including $y<2+5\ln 3$ or $\left(-\infty, 2+5\ln 3\right)$ but not $\left[-\infty, 2+5\ln 3\right]$ as requires a strict inequality at the upper bound. Do not allow $\text{f}<2+5\ln 3$ or $\text{g}<2+5\ln 3$

Question Number	Scheme	Marks
6 (a)	$y=(4x-7)^{\frac{1}{2}} \Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right) 2(4x-7)^{-\frac{1}{2}}$ (see notes)	M1 A1
	At $(8,5)$ gradient of tangent is $2(4\times 8-7)^{-\frac{1}{2}}\left(=\frac{2}{5}\right)$	dM1
	Equation for l is $y-5=-\frac{5}{2}(x-8)$	ddM1
	$2y-10=-5x+40 \Rightarrow 5x+2y-50=0$ *	A1*
		(5)
(b)	$\int (4x-7)^{\frac{1}{2}}\,\mathrm{d}x=\left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]$ (see notes)	M1, A1
	Complete area = $\int_{\frac{7}{4}}^{8}(4x-7)^{\frac{1}{2}}\,\mathrm{d}x=\left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]_{\frac{7}{4}}^{8}+5$	dM1
	$=\frac{155}{6}$	A1
		(4)
		(9 marks)

(a)

M1: Attempts to differentiate to achieve $a(4x\pm 7)^{-\frac{1}{2}}$ or equivalent. The index does not need to be processed.

Alternatively, attempts to differentiate implicitly $y^2=4x-7 \Rightarrow Ay\frac{\mathrm{d}y}{\mathrm{d}x}=B$

A1: Achieves $2(4x-7)^{-\frac{1}{2}}$ o.e. (which may be unsimplified but the index processed) or using implicit differentiation achieves $2y\frac{\mathrm{d}y}{\mathrm{d}x}=4$

dM1: Substitutes $x = 8$ into their $a(4x\pm 7)^{-\frac{1}{2}}$ or substitutes $y = 5$ into their $Ay\frac{\mathrm{d}y}{\mathrm{d}x}=B$. It does not need to be evaluated. It is dependent on the first method mark but allow this mark to be scored if they have incorrectly manipulated their derivative before substituting in, or made transcription errors including if they lose the $-\frac{1}{2}$ index from the bracket.

ddM1: A full method for the equation of *l*. Look for

- substitution of $x = 8$ (or $y=5$ if implicit differentiation) into an attempt at a derivative
- the application of the negative reciprocal rule
- the use of $(8,5)$ with a correct gradient for their value of *m* to form an equation for the normal.

$y-5="-\frac{5}{2}"(x-8)$ with the coordinates in the correct positions. If $y=-\frac{1}{m}x+c$ is used they must proceed as far as $c=...$

It is dependent on both of the previous two method marks.