Question Number	Scheme	Marks
7(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-\frac{16}{3}(3x-k)^{-2}$	M1A1
		(2)
(b)	$-\frac{16}{3}(3x-k)^{-2}=-12\Rightarrow\left(3x-k\right)^{2}=...$	M1
	$3-k="\pm\frac{2}{3}"\Rightarrow k=...$	**d**M1
	$k=\frac{7}{3}\ ,\ \frac{11}{3}$	A1
		(3)
(c)	$y=\frac{16}{9\left(3(1)-\frac{7}{3}\right)}$	M1 (B1 on ePEN)
	$y-"\frac{8}{3}"=\frac{1}{12}(x-1)$	**d**M1
	$12y-x-31=0$	A1
		(3)
(d)	$\int\frac{16}{9(3x-k)}\mathrm{d}x=\frac{16}{27}\left[\ln(3x-k)\right]$	M1
	$=\frac{16}{27}\left[\ln\left(3x-\frac{7}{3}\right)\right]$	A1ft
	$\frac{16}{27}\left[\ln\left(3x-\frac{7}{3}\right)\right]_{1}^{3}=\frac{16}{27}\left(\ln\left(3(3)-\frac{7}{3}\right)-\ln\left(3(1)-\frac{7}{3}\right)\right)$	**d**M1
	$=\frac{16}{27}\ln(10)$	A1
		(4)
		(12 marks)

(a)

M1: Attempts to differentiate to the form $A(3x-k)^{-2}$ oe e.g. $A(27x-9k)^{-2}$

A1: $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)-\frac{16}{3}(3x-k)^{-2}$ oe

e.g. $-\frac{16}{3(3x-k)^{2}}$ or $-\frac{16}{3(9x^{2}-6kx+k^{2})}$ or $-\frac{16}{27x^{2}-18kx+3k^{2}}$

but **not**

$-432(27x-9k)^{-2}$ or $-\frac{432}{(27x-9k)^{2}}$

as there is a common factor in the numerator and denominator.

(b)

M1: Sets their derivative of the form $A(3x-k)^{-2}$ (or equivalent) equal to -12 (not 12) and rearranges to $...(3x-k)^2=...$ or equivalent e.g. $...(27x-9k)^2=....$ or $...(9x^2-6kx+k^2)=...$

Condone poor squaring e.g. allow $...(9x^2+k^2)=...$

May have already substituted $x = 1$

dM1: **<u>Depends on having obtained $A < 0$</u> (otherwise the equation has no real solutions):**
Substitutes $x = 1$ and solves to find 2 values for k.

If the $(3x-k)^2$ is expanded then the usual rules apply for solving a 3TQ and allow using a calculator. FYI correct 3TQ is $9k^2-54k+77=0$

Depends on the previous method mark.

A1: Achieves $k=\frac{7}{3}$ and $k=\frac{11}{3}$ from a correct method. Accept equivalent exact fractions or recurring decimals $2.\dot{3}$, $3.\dot{6}$ but not rounded decimals e.g. 2.33, 3.67

(c) Note we are scoring the first mark as an M mark not a B mark.

M1(B1 on ePEN):
Uses a value of k from part (b) (or a 'made up' k) and $x = 1$ to find the value of y at P.

dM1: Attempts to find the equation of the normal using their y-coordinate with the gradient $\frac{1}{12}$.

If they use $y=mx+c$ they must proceed as far as $c=...$

Depends on the previous method mark.

A1: $12y-x-31=0$ or any equivalent integer multiple of this equation.

Note if $k=\frac{11}{3}$ is used they should get $x - 12y - 33 = 0$ and would generally score 110

(d)

M1: Integrates to the form $B\ln(3x-k)$ oe e.g. $B\ln\alpha(3x-k)$

A1ft: $\frac{16}{27}\left[\ln\left(3x-\frac{7}{3}\right)\right]$ which may be unsimplified and isw once correct integration is seen.

Follow through their k and allow the letter k and allow if their k is not less than 3.
Ignore any reference to $+c$.
You may need to check their integration carefully.

E.g. $\frac{16}{27}\left[\ln(27x-21)\right]$ is also correct (for $k=\frac{7}{3}$)

Ignore any spurious integral signs after a correct integral is seen.

dM1: Substitutes in the limits 3 and 1 and subtracts either way round. Must have a numeric *k* now.

It is dependent on the first method mark.

A1: $\frac{16}{27}\ln(10)$ or exact equivalent e.g. $\frac{32}{54}\ln(10)$

Use of an incorrect *k* in (d) scores a maximum of M1A1ft dM1A0

Note that in part (d), some candidates may use substitution e.g.

$$u = 3x - k \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 3 \Rightarrow \int \frac{16}{9(3x-k)}\,\mathrm{d}x = \frac{16}{9}\int \frac{1}{u}\,\frac{1}{3}\,\mathrm{d}u = \frac{16}{27}\ln u$$

Score M1 for integrating to the correct form e.g. $k\ln u$

and A1 for $\frac{16}{27}\ln u$ **following through their *k* or the letter *k* as above**

then dM1 for

$$\left[\frac{16}{27}\ln u\right]_{\frac{2}{3}}^{\frac{20}{3}} = \frac{16}{27}\ln\frac{20}{3} - \frac{16}{27}\ln\frac{2}{3} \text{ or } \left[\frac{16}{27}\ln\left(3x-\frac{7}{3}\right)\right]_{1}^{3} = \frac{16}{27}\ln\frac{20}{3} - \frac{16}{27}\ln\frac{2}{3}$$

i.e. applies the correct changed limits or reverts to *x* and uses 3 and 1

$$= \frac{16}{27}\ln(10) \quad \textbf{A1}$$

Note if $k = \frac{11}{3}$ is used they should get $= \frac{16}{27}\ln(8)$ and would generally score 1110

Note:

If you see any responses where the denominator in part (a) is expanded incorrectly

e.g. $\frac{16}{9(3x-k)} = \frac{16}{27x-k}$

and candidates persist with this incorrect expansion then send to review.

A typical response with an expanded denominator:

Question Number	Scheme	Marks
7(a)	$y=\frac{16}{9(3x-k)}=\frac{16}{27x-9k}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{432}{(27x-9k)^2}$	M1A0
(b)	$-\frac{432}{(27x-9k)^2}=-12\Rightarrow 12(27x-9k)^2=432$	M1
	$12(27x-9k)^2=432\Rightarrow(27x-9k)^2=36\Rightarrow 27-9k=\pm 6\Rightarrow k=...$	**d**M1
	$k=\frac{7}{3},\frac{11}{3}$	A1
(c)	$y=\frac{16}{27-21}$	M1 (B1 on ePEN)
	$y-"\frac{8}{3}"=\frac{1}{12}(x-1)$	**d**M1
	$12y-x-31=0$	A1
(d)	$\int\frac{16}{27x-9k}\mathrm{d}x=\frac{16}{27}\left[\ln(27x-9k)\right]$	M1
	$=\frac{16}{27}\left[\ln(27x-21)\right]$	A1ft
	$\frac{16}{27}\left[\ln(27x-21)\right]_1^3=\frac{16}{27}\left(\ln(27(3)-21)-\ln(27-21)\right)$	**d**M1
	$=\frac{16}{27}\ln(10)$	A1

Question Number	Scheme	Marks
8(a)(i)	$\left(\frac{b}{2}, a\right)$	B1B1
(ii)	$(0, a-b)$	B1
(iii)	$\left(\frac{b-a}{2}, 0\right)$ and $\left(\frac{a+b}{2}, 0\right)$	B1B1
		(5)

(a) (i)

B1: One correct coordinate $x = \frac{1}{2}b$ or $y = a$

B1: $\left(\frac{b}{2}, a\right)$ or $x = \frac{1}{2}b$ and $y = a$

May be seen on the sketch.

(ii)

B1: $(0, a-b)$ **or** $x = 0$ and $y = a - b$ **or** just $y = a - b$ without the $x = 0$

May be seen on the sketch and if so, allow just $a - b$ marked in the correct place.

If it is on the sketch, condone $(a-b, 0)$ marked in the **correct place**.

If more than 1 point is offered and no clear decision is made then score B0

(iii)

B1: $\left(\frac{b-a}{2}, 0\right)$ **or** $\left(\frac{a+b}{2}, 0\right)$ Allow just $x = \frac{b-a}{2}$ or $x = \frac{a+b}{2}$ without the $y = 0$

May be seen on the sketch and allow just $\frac{b-a}{2}$ or $\frac{a+b}{2}$ marked in the **correct place**.

If they are on the sketch, condone $\left(0, \frac{b-a}{2}\right)$ or $\left(0, \frac{a+b}{2}\right)$ marked in the **correct place**.

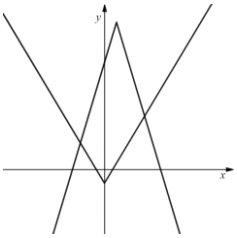
Allow equivalent expressions e.g. $\frac{-a-b}{-2}$ for $\frac{a+b}{2}$ or e.g. $\frac{a-b}{-2}$ for $\frac{b-a}{2}$

B1: $\left(\frac{b-a}{2}, 0\right)$ **and** $\left(\frac{a+b}{2}, 0\right)$ Allow just $x = \frac{b-a}{2}$ or $x = \frac{a+b}{2}$ without the $y = 0$

May be seen on the sketch and allow just $\frac{b-a}{2}$ and $\frac{a+b}{2}$ marked in the **correct place**.

If they are on the sketch, condone $\left(0, \frac{b-a}{2}\right)$ and $\left(0, \frac{a+b}{2}\right)$ marked in the **correct place**.

Allow equivalent expressions e.g. $\frac{-a-b}{-2}$ for $\frac{a+b}{2}$ or e.g. $\frac{a-b}{-2}$ for $\frac{b-a}{2}$

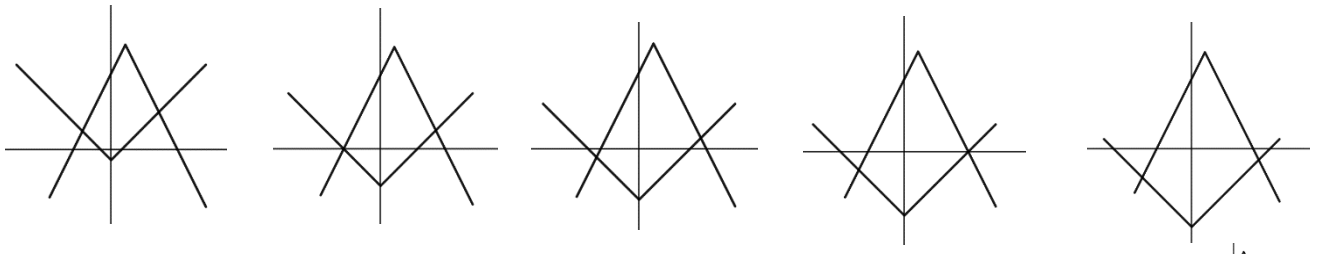
(b)		B1B1
		(2)
(c)	$-x-1=2x+a-b,\ x=-3\Rightarrow 2=-6+a-b$ or $x-1=a+b-2x,\ x=5\Rightarrow 5-1=a+b-10$	M1
	$-x-1=2x+a-b,\ x=-3\Rightarrow 2=-6+a-b$ and $x-1=a+b-2x,\ x=5\Rightarrow 5-1=a+b-10$	dM1 (A1 on ePEN)
	$\begin{matrix}a-b=8\\a+b=14\end{matrix}\Rightarrow a=...$ or $b=...$	**dd**M1
	$a=11,\ b=3$	A1
		(4)
		(11 marks)

(b)

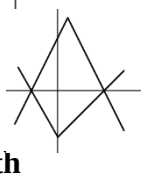
B1: A V shaped graph anywhere on the set of axes

B1: Correct shape with a minimum point on the *y*-axis below the *x*-axis and with an intention for the graph to be symmetrical about the *y*-axis. The left branch of $y=|x|-1$ must intersect the left branch of $y=a-|2x-b|$ and the right branch of $y=|x|-1$ must intersect the right branch of $y=a-|2x-b|$

These are all acceptable for both marks:



But the intersections of the branches must not both be on the *x*-axis e.g.



Some candidates may use Figure 2 to do their sketch which is acceptable but if both Diagram 1 and Figure 2 are used and neither is "rejected" then Diagram 1 takes precedence.

If they draw a sketch of $y=|x|-1$ on their own axes, only the first B mark is available unless they draw the given graph as well in which case the 2nd B is available.