Question Number	Scheme	Marks
4(a)	$\mathrm{f}(x)=\dfrac{2x^2-32}{(3x-5)(x+4)}+\dfrac{8}{3x-5}$	B1
	$=\dfrac{2x^2-32}{(3x-5)(x+4)}+\dfrac{8}{3x-5}=\dfrac{2x^2-32+8(x+4)}{(3x-5)(x+4)}$ or e.g. $=\dfrac{2(x-4)(x+4)}{(3x-5)(x+4)}+\dfrac{8}{3x-5}=\dfrac{2(x-4)+8}{(3x-5)}$	M1
	$=\dfrac{2x\cancel{(x+4)}}{(3x-5)\cancel{(x+4)}}=\dfrac{2x}{3x-5}$ *	A1*
		(3)
(b)	$\mathrm{f}'(x)=\dfrac{2(3x-5)-3\times 2x}{(3x-5)^2}$	M1A1
	$\mathrm{f}'(x)=\dfrac{-10}{(3x-5)^2}$ As $(3x-5)^2>0$ (for $x>2$) then $\mathrm{f}'(x)<0$ Hence f is a decreasing function *	A1cso*
		(3)
(c)	$\mathrm{g}^{-1}(x)=\mathrm{e}^{\frac{x-3}{2}}$	M1A1
	$x\geqslant 3$	B1
		(3)
(d)	$\mathrm{g}\left(\dfrac{2a}{3a-5}\right)=5\Rightarrow 3+2\ln\left(\dfrac{2a}{3a-5}\right)=5$	B1
	$\ln\left(\dfrac{2a}{3a-5}\right)=1\Rightarrow\dfrac{2a}{3a-5}=\mathrm{e}$	M1
	$\dfrac{2a}{3a-5}=\mathrm{e}\Rightarrow a=...$	**d**M1
	$a=\dfrac{5\mathrm{e}}{3\mathrm{e}-2}$	A1
		(4)
(d) Way 2	$\mathrm{gf}(a)=5\Rightarrow\dfrac{2a}{3a-5}=\mathrm{g}^{-1}(5)$	B1
	$\dfrac{2a}{3a-5}=\mathrm{g}^{-1}(5)\Rightarrow\dfrac{2a}{3a-5}=\mathrm{e}$	M1
	$\dfrac{2a}{3a-5}=\mathrm{e}\Rightarrow a=...$	**d**M1
	$a=\dfrac{5\mathrm{e}}{3\mathrm{e}-2}$	A1
		(13 marks)

(a)

B1: $3x^2+7x-20=(3x-5)(x+4)$ seen or used

M1: Combines fractions with a correct common denominator and the order of terms in the numerator consistent with their common denominator.

A1*: Achieves the given answer with no errors seen but condone e.g. a missing trailing bracket as long as it is "recovered".

Sufficient working should be shown but allow to go from e.g. $\frac{2(x-4)+8}{3x-5}$ to $\frac{2x}{3x-5}$

Note that candidates may take a longer route:

$$\frac{2x^2-32}{3x^2+7x-20}+\frac{8}{3x-5}=\frac{(2x^2-32)(3x-5)+8(3x^2+7x-20)}{(3x^2+7x-20)(3x-5)}$$ **M1**

$$=\frac{6x^3+14x^2-40x}{(3x^2+7x-20)(3x-5)}=\frac{2x(3x^2+7x-20)}{(3x-5)(x+4)(3x-5)}$$ **B1**

$$=\frac{2x(3x-5)(x+4)}{(3x-5)(x+4)(3x-5)}=\frac{2x}{3x-5}*$$ **A1***

Or

$$\frac{2x^2-32}{3x^2+7x-20}+\frac{8}{3x-5}=\frac{(2x^2-32)(3x-5)+8(3x^2+7x-20)}{(3x^2+7x-20)(3x-5)}$$ **M1**

$$=\frac{6x^3+14x^2-40x}{(3x^2+7x-20)(3x-5)}=\frac{2x(3x^2+7x-20)}{(3x^2+7x-20)(3x-5)}$$ **B1**

In this case the B1 is scored for obtaining a factor of $3x^2+7x-20$ in the numerator and denominator

$$=\frac{2x}{3x-5}*$$ **A1***

(b)

M1: Attempts to differentiate using the product or quotient rule. Award for an expression of the form $\frac{\alpha(3x-5)-\beta x}{(3x-5)^2}$ or $\alpha(3x-5)^{-1}-\beta x(3x-5)^{-2},\ \alpha,\beta>0$

Condone attempts where $(3x-5)^2$ is expanded.

Alternatively, attempts to write $\frac{2x}{3x-5}$ as $A+\frac{B}{3x-5}$ before differentiating using the chain rule to obtain $\frac{\pm k}{(3x-5)^2}$

A1: Correct derivative simplified or unsimplified.

A1cso*: Requires a correct simplified derivative $\frac{-10}{(3x-5)^2}$ and then statements that convey:

- $(3x-5)^2>0$ oe e.g. denominator is positive (condone $(3x-5)^2\geqslant 0$)
- so $f'(x)<0$
- function is decreasing

Some candidates may attempt to differentiate the original f(x) e.g.:

$$\mathrm{f}'(x)=\frac{4x\left(3x^2+7x-20\right)-\left(2x^2-32\right)(6x+7)}{(3x^2+7x-20)^2}-\frac{24}{(3x-5)^2}$$

Score M1 for:

$$\mathrm{f}'(x)=\frac{Ax\left(3x^2+7x-20\right)-\left(2x^2-32\right)(Cx+D)}{(3x^2+7x-20)^2}-\frac{E}{(3x-5)^2}$$

and (<u>first</u>) A1 if they reach $\dfrac{-10}{(3x-5)^2}$

(c)

M1: Rearranges $y=3+2\ln x$ to $x=\mathrm{e}^{\mathrm{f}(y)}$ or $x=3+2\ln y$ to $y=\mathrm{e}^{\mathrm{f}(x)}$

A1: Obtains $\mathrm{g}^{-1}(x)=\mathrm{e}^{\frac{x-3}{2}}$ or equivalent e.g. $\mathrm{g}^{-1}(x)=\sqrt{\mathrm{e}^{x-3}}$, $\mathrm{g}^{-1}(x)=\mathrm{e}^{\frac{3-x}{-2}}$ and isw.
Accept $\mathrm{g}^{-1}(x)=...$, $\mathrm{g}^{-1}=...$, $y=...$ but not e.g. $\mathrm{f}^{-1}(x)=...$

B1: Correct domain: $x\geqslant 3$ oe using correct notation e.g. $[3,\infty)$

(d) Condone the miscopy of f(x) for the M marks as long as it has the correct form e.g. $\dfrac{...a}{...a\pm...}$

and allow use of *x* instead of *a* for <u>all</u> marks in (d)

B1: Sets up a valid equation in a e.g. $3+2\ln\left(\dfrac{2a}{3a-5}\right)=5$

M1: Rearranges to obtain $\ln\left(\dfrac{2a}{3a-5}\right)=k$ and uses the inverse law of logarithms to remove the ln correctly to reach $\dfrac{2a}{3a-5}=\mathrm{e}^k$ oe

dM1: Rearranges an equation of the form $\dfrac{2a}{3a-5}=\mathrm{e}^k$ to obtain $a=\dfrac{...\mathrm{e}^k}{...\mathrm{e}^k\pm...}$ or equivalent.
Depends on the previous method mark.

A1: $(a=)\ \dfrac{5\mathrm{e}}{3\mathrm{e}-2}$ oe e.g. $(a=)\ \dfrac{-5\mathrm{e}}{2-3\mathrm{e}}$ and apply isw once the correct exact answer is seen.
Accept e^1 for e.

(d) Way 2

B1: Sets up a valid equation in a e.g. $\frac{2a}{3a-5}=\text{g}^{-1}(5)$ seen or implied.

M1: Attempts $\text{g}^{-1}(5)$ and reaches $\frac{2a}{3a-5}=\text{e}^{k}$ oe

dM1: Rearranges an equation of the form $\frac{2a}{3a-5}=\text{e}^{k}$ to obtain $a=\frac{...\text{e}^{k}}{...\text{e}^{k}\pm...}$ or equivalent.

Depends on the previous method mark.

A1: $(a=)\ \frac{5\text{e}}{3\text{e}-2}$ oe e.g. $(a=)\ \frac{-5\text{e}}{2-3\text{e}}$ and apply isw once the correct exact answer is seen.

Accept e^{1} for e.

Alternative for (d) which has been seen:

$$\text{g}\left(\frac{2a}{3a-5}\right)=5\Rightarrow 3+2\ln\left(\frac{2a}{3a-5}\right)=5:\ \textbf{B1 as above}$$

$$\Rightarrow \ln\left(\frac{2a}{3a-5}\right)^{2}=2\Rightarrow\left(\frac{2a}{3a-5}\right)^{2}=\text{e}^{2}:\ \textbf{M1 rearranges to reach } \left(\frac{2a}{3a-5}\right)^{2}=\text{e}^{...}$$

$$\Rightarrow\left(9\text{e}^{2}-4\right)a^{2}-30\text{e}^{2}a+25\text{e}^{2}=0$$

$$\Rightarrow a=\frac{30\text{e}^{2}\pm\sqrt{900\text{e}^{4}-100\text{e}^{2}\left(9\text{e}^{2}-4\right)}}{2\left(9\text{e}^{2}-4\right)}:\ \textbf{dM1 forms and solves 3TQ in } \boldsymbol{a} \textbf{ (usual rules)}$$

$$=\frac{5\text{e}}{3\text{e}-2}\ \text{oe}:\ \textbf{A1 as above}$$

Question Number	Scheme	Marks
5(a)	$(A=)8$	B1 **(1)**
(b)	$16=10+"8"\mathrm{e}^{-Bt}\Rightarrow \mathrm{e}^{-Bt}=...$	M1
	$\mathrm{e}^{-45B}="\frac{3}{4}"\Rightarrow -45B=\ln"\frac{3}{4}"\Rightarrow B=...$	**d**M1
	$B=$ awrt 0.00639	A1
		(3)
(c)	$\frac{\mathrm{d}T}{\mathrm{d}t}=-"8"\times\frac{1}{45}\ln\left(\frac{4}{3}\right)\times\mathrm{e}^{-\frac{1}{45}\ln\left(\frac{4}{3}\right)\times 2}=...$	M1
	$=-0.0505$	A1
		(2)
(d)	The temperature has a (lower) limit of $10\,°\mathrm{C}$ **or** $5=10+"8"\mathrm{e}^{-Bt}\Rightarrow \mathrm{e}^{-Bt}=-\frac{5}{"8"}$ e.g. which is not possible or cannot be solved or you cannot find the log of a negative number	B1 **(1)**
		(7 marks)

(a)

B1: $(A=)8$ or award for $(T=)10+8\mathrm{e}^{-Bt}$. Note that "$A=$" is not required, just look for 8.

(b)

M1: Sets $16=10+"8"\mathrm{e}^{-Bt}$ and rearranges to the form $\mathrm{e}^{-Bt}=...$ with or without $t = 45$

dM1: Uses $t = 45$, takes lns of both sides and proceeds to find a numerical expression for B

Depends on the previous method mark.

A1: Achieves awrt 0.00639 or 6.39×10^{-3} or e.g. $\frac{1}{45}\ln\left(\frac{4}{3}\right)$ or $-\frac{1}{45}\ln\left(\frac{3}{4}\right)$ or $\frac{-\ln(0.75)}{45}$ etc.

and apply isw once the correct value is seen. **Correct answer only scores <u>no</u> marks.**

(c)

M1: Differentiates using the chain rule to obtain $...\mathrm{e}^{-Bt}$ (not $...\mathrm{e}^{Bt}$) where … is a constant

and substitutes in $t=2$ to obtain a value for $\frac{\mathrm{d}T}{\mathrm{d}t}$.

Condone $\frac{\mathrm{d}T}{\mathrm{d}t}="8"\mathrm{e}^{-Bt}$ as long as it is clear they think they have found $\frac{\mathrm{d}T}{\mathrm{d}t}$.

If they lose the minus sign in $...\mathrm{e}^{-Bt}$ they obtain $\pm0.0518…$ and this scores M0

Note that if there is evidence that $\frac{\mathrm{d}T}{\mathrm{d}t}=...t\mathrm{e}^{-Bt}$ is being used, score M0

A1: -0.0505 cao. Accept 0.0505 **decreasing** or equivalent **<u>following correct work</u>**.

Ignore any units, correct or incorrect)

Correct answer only scores <u>no</u> marks.