4. The function f is defined by

$$f(x) = \frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \qquad x \in \mathbb{R} \quad x > 2$$

(a) Show that $f(x) = \dfrac{2x}{3x-5}$

(3)

(b) Show, using calculus, that f is a decreasing function.
You must make your reasoning clear.

(3)

The function g is defined by

$$g(x) = 3 + 2\ln x \qquad x \geqslant 1$$

(c) Find g^{-1}

(3)

(d) Find the exact value of a for which

$$gf(a) = 5$$

(4)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 13 marks)

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, T °C, of the air in a room t minutes after a heat source is switched off, is modelled by the equation

$$T = 10 + Ae^{-Bt}$$

where A and B are constants.

Given that the temperature of the air in the room at the instant the heat source was switched off was 18 °C,

(a) find the value of A

(1)

Given also that, exactly 45 minutes after the heat source was switched off, the temperature of the air in the room was 16 °C,

(b) find the value of B to 3 significant figures.

(3)

Using the values for A and B,

(c) find, according to the model, the rate of change of the temperature of the air in the room exactly two minutes after the heat source was switched off.
Give your answer in °C min^{-1} to 3 significant figures.

(2)

(d) Explain why, according to the model, the temperature of the air in the room cannot fall to 5 °C

(1)

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