Question Number	Scheme	Marks
8(a)	Starting with the LHS: $2\text{cosec}^2 2\theta(1-\cos 2\theta) = \dfrac{2-2\cos 2\theta}{\sin^2 2\theta}$	M1
	$= \dfrac{2-2(1-2\sin^2\theta)}{4\sin^2\theta\cos^2\theta}$	M1dM1
	$= \sec^2\theta = 1+\tan^2\theta \equiv \text{RHS}$ *	A1*
		(4)
(b)	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = ...$	M1
	$\cos x = \dfrac{1}{4}$ (ignore -1)	A1
	$\cos x = \dfrac{1}{4} \Rightarrow x = ...$	dM1
	$x = 75.5°,\ 284.5°$	A1
		(4)
		(8 marks)

Notes

(a) The most common method and where to award marks is as follows:

M1: Uses $\text{cosec}\, 2\theta = \dfrac{1}{\sin 2\theta}$ (oe) at some stage in the proof to convert the cosec into sine.

M1 Attempts to use one of $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ or $\sin 2\theta = 2\sin\theta\cos\theta$ but condone $\sin^2 2\theta = 2\sin^2\theta\cos^2\theta$ as an attempt at squaring.

dM1: Depends on previous M. Attempts to use both $\cos 2\theta = 1-2\sin^2\theta$ oe and $\sin 2\theta = 2\sin\theta\cos\theta$

A1*: Achieves the RHS with no mathematical errors seen and all stages of working shown. Must see the $\sec^2\theta$ before the final answer. Condone minor notational errors (e.g. a missing θ) but not consistent poor notation throughout. There will be other solutions but if they work from both sides then they need a conclusion at the end.

For other methods or variations apply scheme as follows:

M1: Uses a correct identity for either cosec 2θ or $\text{cosec}^2 2\theta$ or $\cot^2 2\theta$ to get the equation in terms of sin and cosine only or to introduce $\text{cosec} 2\theta$ if working in reverse. Condone the 2 becoming ½.

M1: Attempts to apply a double angle identity correct up to sign errors, e.g. $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$

dM1: Depends on previous M. Use of at least two **correct** double angle identities and/or Pythagorean identities to *progress sufficiently towards the goal* (e.g. reduces all terms to single argument form in cosine or sine only, or identify an appropriate factor to cancel to such, or to get all double arguments if working in reverse with the sin 2θ identified). There may be algebraic slips, but all trig identities used up to this point must be correct (e.g. see stag reach in main scheme).

A1*: Fully correct proof with sufficient stages shown including use of $\sec^2\theta = 1+\tan^2\theta$ or equivalent identity (e.g. other Pythagorean relation), and if necessary, a conclusion given.

For example $1+\tan^2\theta = \sec^2\theta = \dfrac{1}{\cos^2\theta} = \dfrac{2}{1+\cos 2\theta}$ 2nd M1, use of a suitable double angle identity

$= \dfrac{2(1-\cos 2\theta)}{(1+\cos 2\theta)(1-\cos 2\theta)} = \dfrac{2(1-\cos 2\theta)}{1-\cos^2 2\theta} = \dfrac{2(1-\cos 2\theta)}{\sin^2 2\theta}$ 3rd dM1, correct initial double angles identity and Pythagorean identity used to identify the sin in the denominator, all identities correct

$= 2\text{cosec}^2 2\theta(1-\cos 2\theta)$ 1st M1, use of correct identity for $\text{cosec}^2 2\theta$. (A1 if all correct)

(b) Note: allow with x or θ and allow recovery of mixed variables in this part.

M1: Uses part (a) to forms a 3 term quadratic in $\sec x$ or cos x, solves their quadratic and finds cos x =… or sec x =… for at least one solution. For reference the quadratic in cos is $4\cos^2 x + 3\cos x - 1 = 0$

A1: $\cos x = \frac{1}{4}$ (ignore any reference to $\cos x = -1$) Must have a correct cosine or implied by a correct answer after reaching a secant if no incorrect working seen. May be implied if not seen explicitly.

dM1: Correct method to find a value for x from cos $x = k$ where $|k| < 1$ as a solution for their equation. May be implied by one correct angle from $\sec x = k$ or $\cos x = k$. Note radian answer awrt 1.3 implies dM1.

A1: awrt $75.5°$ and awrt $284.5°$ Allow with or without 180° included, but there must be no other angles in given interval. All previous marks must have been scored. Ignore answers outside the domain.

(b)Alt	$1+\tan^2 x = 4 + 3\sec x \Rightarrow 9\sec^2 x = \tan^4 x - 6\tan^2 x + 9 \Rightarrow \tan^4 x - 15\tan^2 x = 0$ $\Rightarrow \tan^2 x = ...(\neq 0)$	M1
	$\tan x = (\pm)\sqrt{15}$	A1
	$\tan x = k \neq 0 \Rightarrow x = ...$	dM1
	$x = 75.5°, 284.5°$	A1
		(4)

Notes

(b) Alt

M1: Makes sec x the subject, squares and solves the resulting quadratic in $\tan^2 x$ leading to a value for $\tan^2 x$. Note there may be variations on this approach, such as $1+\tan^2 x = 4 + 3\sqrt{1+\tan^2 x}$ being used before squaring, or solving for "$1+\tan^2 x$". Score for a correct approach leading to a value for $\tan^2 x$.

A1: Correct value for tan x. Need not give both values.

dM1: Solves from their tan $x = k$, $k \neq 0$ to find a value for x

A1: Both correct values obtained and no invalid solutions in the range (ignore 180° as main scheme). Must reject extra solutions.

Note: other methods or variations may be seen, which can be marked according to the same principles, first M for correct approach to find a value for a trig ratio, A1 correct non-trivial value, dM1 solves for x.

Question Number	Scheme	Marks
9(a)	$k=-1$	B1
		(1)
(b)(i)	$\mathrm{f}(0)=\lvert 2-4\ln(0+1)\rvert=2-0=2$	B1
(ii)	$0=2-4\ln(x+1)\Rightarrow \ln(x+1)=\dfrac{1}{2}\Rightarrow x=\mathrm{e}^{p}+q$	M1
	$x=\mathrm{e}^{\frac{1}{2}}-1$	A1
		(3)
(c)	$2-4\ln(x+1)=3\Rightarrow \ln(x+1)=...$ or $-2+4\ln(x+1)=3\Rightarrow \ln(x+1)=...$	M1
	$2-4\ln(x+1)=3\Rightarrow x=...$ and $-2+4\ln(x+1)=3\Rightarrow x=...$	dM1
	CVs $\mathrm{e}^{-\frac{1}{4}}-1,\ \mathrm{e}^{\frac{5}{4}}-1$	A1
	$"-1"<x<\mathrm{e}^{-\frac{1}{4}}-1$ or $x>\mathrm{e}^{\frac{5}{4}}-1$	ddM1A1ft
		(5)
		(9 marks)

Notes

(a)

B1: For $(k=)-1$. Accept $x=-1$ or even $x>-1$

(b) (i)

B1: For 2 seen as the *y* coordinate. May be identified as *y* or f(0) or (0, 2).

(ii)

M1: Sets $0=2-4\ln(x+1)$, rearranges and proceeds to $x=...$ of the correct form $\mathrm{e}^{p}+q,\ q\neq 0$. May be implied by awrt 0.65.

A1: $x=\mathrm{e}^{\frac{1}{2}}-1$ or accept $\sqrt{\mathrm{e}}-1$ or even $\mathrm{e}^{\frac{2}{4}}-1$ May be seen as part of a coordinate pair.

(c) Note this is M1A1A1M1A1 on epen but is being marked as M1M1A1M1A1.

M1: Forms one valid equation with the modulus signs removed and attempts to solve at least as far as ln(*x*+1) . Accept with > or < etc instead of = for the M.

dM1: Both equations attempted, allowing for "=" or any inequality used, to achieve values (may be decimals) for each via $\ln(x+1)=\ldots$ and suitable attempt to undo ln (raises to a base power, allow an error with the base). For reference the decimals are −0.221 and 2.49 to 3 s.f.

A1: Both $\mathrm{e}^{-\frac{1}{4}}-1,\ \mathrm{e}^{\frac{5}{4}}-1$ Must be exact.

ddM1: Chooses the outside region for their values. Allow with strict or non-strict inequalities. It is dependent on both previous method marks and requires two distinct critical values. The left hand bound −1 may be missing for this mark (i.e. allow for $x<\mathrm{e}^{-\frac{1}{4}}-1$ or $x>\mathrm{e}^{\frac{5}{4}}-1$) Allow with decimal values for this mark. ddM0 if the middle section is also included but isw if they choose the correct outside region but later reject the left hand portion due to confusion with $x>-1$ Watch out for $x>\ldots$ for both inequalities, which is ddM0.

A1ft: $"-1"<x<\mathrm{e}^{-\frac{1}{4}}-1$ or $x>\mathrm{e}^{\frac{5}{4}}-1$ or any equivalent form following through on their **negative** *k* from (a). Must be exact (not decimals). Allow "and" instead of "or" – ie both regions are identified as the solution. But use of $\cap$ in set notation is A0 as it is incorrect.