Question Number	Scheme	Marks
7(a)	$\mathrm{f}(0)=(0-3)^2=9$	M1
	$0 \leqslant \mathrm{f}(x) \leqslant 9$	A1
		(2)
(b)	$\mathrm{f}'(x)=-2x\mathrm{e}^{-x^2}(2x^2-3)^2+\mathrm{e}^{-x^2}\times 8x(2x^2-3)$	M1A1
	$=2x(2x^2-3)\mathrm{e}^{-x^2}\left(-(2x^2-3)+4\right)=2x\mathrm{e}^{-x^2}(2x^2-3)\left(7-2x^2\right)$	dM1A1
		(4)
(c)	$x^2=\frac{3}{2},\frac{7}{2}\Rightarrow \mathrm{f}\left(\sqrt{\frac{7}{2}}\right)=\mathrm{e}^{-\frac{7}{2}}\left(2\times\frac{7}{2}-3\right)^2=16\mathrm{e}^{-\frac{7}{2}}$	M1A1
	$16\mathrm{e}^{-\frac{7}{2}}<k<9$	dM1A1
		(4)
		(10 marks)

Notes

(a) Work for part (a) must be seen in part (a) not recovered in part (c).

M1: Substitutes $x=0$ and proceeds to find a value for *y*. Implied by sight of 9.

A1: $0 \leqslant \mathrm{f}(x) \leqslant 9$ Accept with *y* or just f, but not with *x*. Accept interval, [0, 9], or set notation.

(b)

M1: Attempts the product rule and chain rule achieving $\pm Px\mathrm{e}^{-x^2}(2x^2\pm 3)^2+\mathrm{e}^{-x^2}\times Qx(2x^2\pm 3)$ with *P*, *Q* > 0. Alternatively, attempts the quotient rule on $\frac{(2x^2-3)^2}{\mathrm{e}^{x^2}}$ achieving $\frac{Px(2x^2\pm 3)\mathrm{e}^{x^2}-Qx\mathrm{e}^{x^2}(2x^2\pm 3)^2}{\left(\mathrm{e}^{x^2}\right)^2}$ *P*, *Q* > 0

A1: $-2x\mathrm{e}^{-x^2}(2x^2-3)^2+\mathrm{e}^{-x^2}\times 8x(2x^2-3)$ oe (need not be simplified)

dM1: Must have scored previous M. Attempts to takes out a factor of $2x\mathrm{e}^{-x^2}(2x^2-3)$ to obtain a factor $\left(\pm C\pm Dx^2\right)$ which may be unsimplified. Allow if e.g. the *x* or e^{-x^2} is dropped when taking out the factor.

A1: Achieves $2x\mathrm{e}^{-x^2}(2x^2-3)(7-2x^2)$ with no errors seen.

(c) **Note : allow all the marks in (c) from answers which were a constant multiple out in (b) or missing the factor *x*, which lead to the correct answers here.**

M1: Attempts to find a *y* value (which may be zero) for at least one of the valid non-zero roots for f'(*x*)=0. Allow for an attempt at either $\mathrm{f}\left(\pm\sqrt{\frac{3}{2}}\right)$ or $\mathrm{f}\left(\pm\sqrt{\frac{A}{B}}\right)$ leading to a value, where *A* and *B* are their values from (b) with *AB* > 0. Note e.g. $\mathrm{f}\left(\frac{A}{B}\right)$ attempted is M0.

A1: For obtaining $16\mathrm{e}^{-\frac{7}{2}}$ Accept awrt 0.483 for this mark following the award of M.

dM1: Attempts the inside region, allowing $\leqslant$, using their *y* intercept from (a) and their **positive** *y* value, less than their intercept, from an attempt at the at the *y* value of the maxima. Allow a positive *y* value from an attempt at any non-zero root of f'(*x*) as such an attempt. It is dependent on the previous method mark.

A1: $16\mathrm{e}^{-\frac{7}{2}}<k<9$ or in any equivalent form e.g. interval notation $k\in\left(\frac{16}{\mathrm{e}^{7/2}},9\right)$ Allow *y* instead of *k*

Ignore references to "*k* = 0"

Question Number	Scheme	Marks
8(a)	Starting with the LHS: $2\text{cosec}^2 2\theta(1-\cos 2\theta) = \dfrac{2-2\cos 2\theta}{\sin^2 2\theta}$	M1
	$= \dfrac{2-2(1-2\sin^2\theta)}{4\sin^2\theta\cos^2\theta}$	M1dM1
	$= \sec^2\theta = 1+\tan^2\theta \equiv \text{RHS}$ *	A1*
		(4)
(b)	$\sec^2 x - 3\sec x - 4 = 0 \Rightarrow \sec x = ...$	M1
	$\cos x = \dfrac{1}{4}$ (ignore -1)	A1
	$\cos x = \dfrac{1}{4} \Rightarrow x = ...$	dM1
	$x = 75.5°,\ 284.5°$	A1
		(4)
		(8 marks)

Notes

(a) The most common method and where to award marks is as follows:

M1: Uses $\text{cosec}\, 2\theta = \dfrac{1}{\sin 2\theta}$ (oe) at some stage in the proof to convert the cosec into sine.

M1 Attempts to use one of $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$ or $\sin 2\theta = 2\sin\theta\cos\theta$ but condone $\sin^2 2\theta = 2\sin^2\theta\cos^2\theta$ as an attempt at squaring.

dM1: Depends on previous M. Attempts to use both $\cos 2\theta = 1-2\sin^2\theta$ oe and $\sin 2\theta = 2\sin\theta\cos\theta$

A1*: Achieves the RHS with no mathematical errors seen and all stages of working shown. Must see the $\sec^2\theta$ before the final answer. Condone minor notational errors (e.g. a missing θ) but not consistent poor notation throughout. There will be other solutions but if they work from both sides then they need a conclusion at the end.

For other methods or variations apply scheme as follows:

M1: Uses a correct identity for either cosec 2θ or $\text{cosec}^2 2\theta$ or $\cot^2 2\theta$ to get the equation in terms of sin and cosine only or to introduce $\text{cosec} 2\theta$ if working in reverse. Condone the 2 becoming ½.

M1: Attempts to apply a double angle identity correct up to sign errors, e.g. $\cos 2\theta = \pm 1 \pm 2\sin^2\theta$

dM1: Depends on previous M. Use of at least two **correct** double angle identities and/or Pythagorean identities to *progress sufficiently towards the goal* (e.g. reduces all terms to single argument form in cosine or sine only, or identify an appropriate factor to cancel to such, or to get all double arguments if working in reverse with the sin 2θ identified). There may be algebraic slips, but all trig identities used up to this point must be correct (e.g. see stag reach in main scheme).

A1*: Fully correct proof with sufficient stages shown including use of $\sec^2\theta = 1+\tan^2\theta$ or equivalent identity (e.g. other Pythagorean relation), and if necessary, a conclusion given.

For example $1+\tan^2\theta = \sec^2\theta = \dfrac{1}{\cos^2\theta} = \dfrac{2}{1+\cos 2\theta}$ 2nd M1, use of a suitable double angle identity

$= \dfrac{2(1-\cos 2\theta)}{(1+\cos 2\theta)(1-\cos 2\theta)} = \dfrac{2(1-\cos 2\theta)}{1-\cos^2 2\theta} = \dfrac{2(1-\cos 2\theta)}{\sin^2 2\theta}$ 3rd dM1, correct initial double angles identity and Pythagorean identity used to identify the sin in the denominator, all identities correct

$= 2\text{cosec}^2 2\theta(1-\cos 2\theta)$ 1st M1, use of correct identity for $\text{cosec}^2 2\theta$. (A1 if all correct)