7. A scientist is studying two different populations of bacteria.

The number of bacteria N in the first population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 2500 bacteria in this population at the start of the study
- there were 10 000 bacteria 8 hours later

(a) find the exact value of A and the value of k to 4 significant figures.

(3)

The number of bacteria N in the second population is modelled by the equation

$$N = 60\,000e^{-0.6t} \qquad t \geqslant 0$$

where t is the time in hours from the start of the study.

(b) Find the rate of decrease of bacteria in this population exactly 5 hours from the start of the study. Give your answer to 3 significant figures.

(2)

When $t = T$, the number of bacteria in the two different populations was the same.

(c) Find the value of T, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable*.)

(3)

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8.

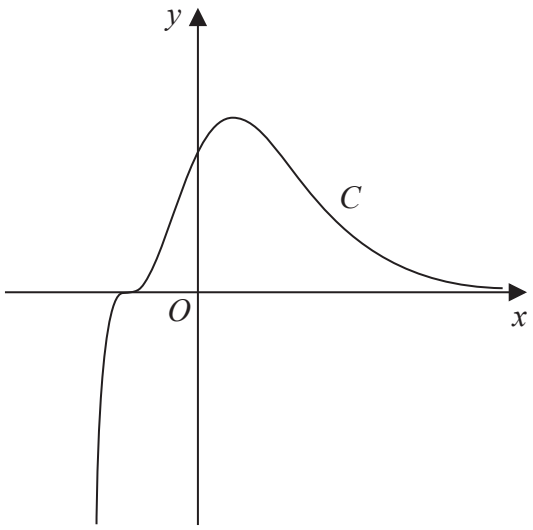


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (2x + 1)^3 \mathrm{e}^{-4x}$$

(a) Show that

$$\mathrm{f}'(x) = A(2x + 1)^2 (1 - 4x)\, \mathrm{e}^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C.

(3)

The function g is defined by

$$\mathrm{g}(x) = 8\,\mathrm{f}(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = \mathrm{g}(x)$.

(2)

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