Question Number	Scheme	Marks
9(a)	$\frac{\cos 2x}{\sin x}+\frac{\sin 2x}{\cos x}=\frac{\cos 2x}{\sin x}+\frac{2\sin x\cos x}{\cos x}$ (One Correct identity)	B1
	$=\frac{1-2\sin^2 x}{\sin x}+\frac{2\sin x\cancel{\cos x}}{\cancel{\cos x}}$	M1
	$=\frac{1}{\sin x}-\frac{2\sin\cancel{^2}x}{\cancel{\sin x}}+2\sin x=\frac{1}{\sin x}=\text{cosec } x$ *	A1*
		(3)
(b)	E.g. Equation is $\text{cosec}^2\theta=6\cot\theta-4\Rightarrow 1+\cot^2\theta=6\cot\theta-4$	M1
	E.g. $\cot^2\theta-6\cot\theta+5=0$	A1
	E.g. $\tan\theta=\frac{1}{5},1$	dM1
	$\theta=0.197,\frac{\pi}{4}$	A1, A1
		(5)
(c)	$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\text{cosec}x\cot x\,\text{d}x=\left[-\text{cosec}x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	M1
	$=2-\sqrt{2}$	A1
		(2)
		10 marks

Notes

(a) **There are lots of ways of proving this statement. In general score as follows**

B1 For applying at least one CORRECT double or compound angle identity during the proof or for forming a CORRECT single fraction initially.

M1 For a correct overall strategy, e.g. applying double angle identities to reduce terms to single angle arguments and cancelling down terms to eliminate cos *x* terms (score at the stage cos *x* terms could be eliminated), or attempting a single fraction and applying relevant identities to achieve single angle argument with common factor cos *x* in the numerator. Allow slips in signs, such as $\cos 2x=1\pm 2\sin^2 x$ for the M but otherwise identities used must be correct.

A1* Fully correct proof showing all necessary steps, though the left hand side may be implied (and may follow initial lines of aside working). Must see the $\frac{1}{\sin x}\rightarrow \text{cosec } x$ during the proof . Do not penalise minor notational slips such as missing an *x* in one term.

(b)

M1 Correctly applies the result of (a) and attempts to use relevant identities, allowing sign errors e.g. $\pm 1\pm\cot^2\theta=\text{cosec}^2\theta$ to produce an equation in $\cot\theta$ or other single trig term only. An alternative is

$$\frac{1}{\sin^2\theta}=6\frac{\cos\theta}{\sin\theta}-4\Rightarrow 1=6\sin\theta\cos\theta-4\sin^2\theta\Rightarrow\left(1+4\sin^2\theta\right)^2=36\sin^2\theta\left(1-\sin^2\theta\right)$$

A1 Correct quadratic $\cot^2\theta-6\cot\theta+5=0$ or $5\tan^2\theta-6\tan\theta+1=0$. In the alternative, a correct quadratic in $\sin^2\theta$ or $\cos^2\theta$ e.g. $52\sin^4\theta-28\sin^2\theta+1=0$. The "=0" may be implied by an attempt to solve. May be implied by correct solutions following an unsimplified quadratic.

dM1 Attempts to solve quadratic to find at least one value for their trig term used. Usual rules, may use calculator.

A1 One correct value for θ following from a correct value for the trig term they are working in — must have solved a correct quadratic in the dM. Accept awrt 0.197 or 0.785. Degrees answer are A0A0.

Question Number	Scheme	Marks
A1	Both values correct and no other values in the range. Accept awrt 0.197 and $\frac{\pi}{4}$ only (must be exact but isw after correct value seen).	
Note:	Allow if a different variable used (such as x). For mixed variables allow the M's but only allow the first A (and final A's) if recovered.	
Note	Answers without working score no marks.	
(c)		
M1	For using part (a) and achieving $\pm k\,\text{cosec}x$ oe for the integral (limits not required for this mark. May arise from longer methods, but must achieve the correct form.	
A1	$2-\sqrt{2}$ Must have scored the M - answer only with no integration shown is M0A0.	
(a) ALT I	$\frac{\cos 2x}{\sin x}+\frac{\sin 2x}{\cos x}=\frac{\cos 2x\cos x+\sin 2x\sin x}{\sin x\cos x}$ Correct single fraction	B1
	$=\frac{\cos x\left(1-2\sin^2 x\right)+2\sin x\cos x\sin x}{\sin x\cos x}$ Single fraction with single arguments and common factor cos x in numerator	M1
	$=\frac{\cancel{\cos x}}{\sin x\cancel{\cos x}}=\frac{1}{\sin x}=\text{cosec } x$ *	A1* **(3)**
(a) ALT II	$\frac{\cos 2x}{\sin x}+\frac{\sin 2x}{\cos x}\equiv\frac{\cos 2x\cos x+\sin 2x\sin x}{\sin x\cos x}$ Correct single fraction	B1
	$\equiv\frac{\cos\left(2x-x\right)}{\sin x\cos x}$ Applies identity to reach single fraction with single arguments and common factor cos x in numerator	M1
	$\equiv\frac{\cancel{\cos x}}{\sin x\cancel{\cos x}}\equiv\frac{1}{\sin x}\equiv\text{cosec } x$ *	A1* **(3)**
	Note $\cos(x-2x)$ is equally correct for the M1.	
(a) ALT III	$\frac{\cos 2x}{\sin x}+\frac{\sin 2x}{\cos x}\equiv\frac{\cos^2 x-\sin^2 x}{\sin x}+\frac{2\sin x\cos x}{\cos x}$ Correct identity	B1
	$\equiv\frac{\cos^3 x-\sin^2 x\cos x+2\sin^2 x\cos x}{\sin x\cos x}$ Single fraction with single arguments and common factor cos x in numerator	M1
	$\equiv\frac{(\cos^2 x+\sin^2 x)\cancel{\cos x}}{\sin x\cancel{\cos x}}\equiv\frac{1}{\sin x}\equiv\text{cosec } x$ *	A1* **(3)**

Question Number	Scheme	Marks
10 (a)	$x=\dfrac{2y^2+6}{3y-3}\Rightarrow\left(\dfrac{\mathrm{d}x}{\mathrm{d}y}=\right)\dfrac{4y(3y-3)-3(2y^2+6)}{(3y-3)^2}$	M1 A1
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{6y^2-12y-18}{9(y-1)^2}=\dfrac{2y^2-4y-6}{3(y-1)^2}$ o.e	dM1, A1
		(4)
(b)	P and Q are where $\dfrac{\mathrm{d}x}{\mathrm{d}y}=0$ or where $2y^2-4y-6=0$	B1
	Solves $2y^2-4y-6=0\Rightarrow 2(y-3)(y+1)=0\Rightarrow y=3,-1$	M1
	Subs $y=-1$ and 3 in $x=\dfrac{2y^2+6}{3y-3}\Rightarrow x=..$	dM1
	Achieves $x=-\dfrac{4}{3}$ and $x=4$	A1cso
		(4) **8 marks**

Notes

(a)

M1 Attempts the quotient rule. Condone slips on the coefficients - look for $\dfrac{Ay(3y-3)-B(2y^2+6)}{(3y-3)^2}$

$A,B > 0$. Allow a product rule attempt:

$x=(2y^2+6)(3y-3)^{-1}\Rightarrow\left(\dfrac{\mathrm{d}x}{\mathrm{d}y}=\right)Ay(3y-3)^{-1}+(2y^2+6)\times -B(3y-3)^{-2}$

A1 Correct differentiation which may be unsimplified. Allow if the $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ is missing or called $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ for this mark. By product rule $4y(3y-3)^{-1}+(2y^2+6)\times -3(3y-3)^{-2}$ Condone missing brackets if recovered.

dM1 Requires an attempt to get a single fraction with some attempt to simplify.
For the quotient rule look for a simplification of the numerator with like terms collected giving a 3TQ.
Attempts via the product rule will require a correct method to put as a single fraction.

A1 $\left(\dfrac{\mathrm{d}x}{\mathrm{d}y}=\right)\dfrac{2y^2-4y-6}{3y^2-6y+3}$ or exact simplified equivalent such as $\dfrac{2(y-3)(y+1)}{3(y-1)^2}$ isw after a correct simplified answer. Common factor 3 must have been cancelled. Must be seen in part (a). A0 if called $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ but allow A1 if LHS is not stated.

Attempts at $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can score the first 3 marks if correct. Allow use of x in place of y for the Ms.

(b)

B1 Indicates P and Q are where $\dfrac{\mathrm{d}x}{\mathrm{d}y}=0$ or where their $2y^2-4y-6=0$ (which may be the denominator of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ if they found this instead).

M1 Solves their 3TQ from an attempt at $\dfrac{\mathrm{d}x}{\mathrm{d}y}=0$ (or denominator of their $\dfrac{\mathrm{d}y}{\mathrm{d}x}$=0), usual rules.