Question Number	Scheme	Marks
7(a)	$x = 3\tan\left(y - \frac{\pi}{6}\right) \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = 3\sec^2\left(y - \frac{\pi}{6}\right)$	B1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3\sec^2\left(y - \frac{\pi}{6}\right)}$	M1
	$\frac{1}{3\left(1+\tan^2\left(y - \frac{\pi}{6}\right)\right)} = \frac{1}{3\left(1+\left(\frac{x}{3}\right)^2\right)}$	dM1
	$= \frac{3}{x^2+9}$	A1
		(4)
(b)	$y = \frac{\pi}{3} \Rightarrow x = 3\tan\frac{\pi}{6} = \sqrt{3}$	B1
	$x = \sqrt{3} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{\left(\sqrt{3}\right)^2+9}$ or $y = \frac{\pi}{3} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3\sec^2\left(\frac{\pi}{3} - \frac{\pi}{6}\right)} = \ldots$	M1
	$y - \frac{\pi}{3} = \frac{1}{4}\left(x - \sqrt{3}\right)$	dM1
	$y = 0 \Rightarrow 0 - \frac{\pi}{3} = \frac{1}{4}\left(x - \sqrt{3}\right) \Rightarrow x = \ldots$	ddM1
	$x = \sqrt{3} - \frac{4\pi}{3}$	A1
		(5)
		Total 9

(a)

B1: Correct derivative, including correct lhs. Condone $\frac{\mathrm{d}x}{\mathrm{d}y} = k\sec^2\left(y - \frac{\pi}{6}\right)$ where k is a constant.

M1: Either (1) attempts to apply the reciprocal rule $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 \div \frac{\mathrm{d}x}{\mathrm{d}y}$.

Don't be too concerned by the position of the constant, it is the function and variable that is important.

Or (2) attempts to apply $\sec^2\left(y - \frac{\pi}{6}\right) = 1 + \tan^2\left(y - \frac{\pi}{6}\right)$ with $\tan\left(y - \frac{\pi}{6}\right)$ being replaced by $\frac{x}{3}$ to get $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of x

dM1: Attempts both (1) and (2) to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x

A1: Correct answer

(b)

B1: Correct value for x. Condone awrt 1.73. This can be awarded from the sight of this value in an equation

M1: Uses a correct method to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ which may be a decimal.

Note that this can be scored either from answer to part (a) e.g $x = "\sqrt{3}" \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = "\dfrac{3}{\left(\sqrt{3}\right)^2 + 9}"$

Or via their $\dfrac{\mathrm{d}x}{\mathrm{d}y} = 3\sec^2\left(\dfrac{\pi}{3} - \dfrac{\pi}{6}\right)$ followed by an attempt at the reciprocal

dM1: Correct straight line method for the tangent at $\left("\sqrt{3}", \dfrac{\pi}{3}\right)$ using their correctly found *m*.

It is dependent upon having found the gradient and *x* value using a correct method.
If they use the form $y = mx + c$ they must proceed as far as *c* = ...

ddM1: Uses *y* = 0 to find *x*. It is dependent upon having scored the previous mark

A1: Correct value or exact equivalent for example $\dfrac{3\sqrt{3} - 4\pi}{3}$

Alt (a) via arctan

B1: $x = 3\tan\left(y - \dfrac{\pi}{6}\right) \Rightarrow y = \dfrac{\pi}{6} + \arctan\left(\dfrac{x}{3}\right) \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 + \dfrac{1}{1 + \left(\dfrac{x}{\ldots}\right)^2} \times \ldots$ where ... could be any values even 1

M1: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 + \dfrac{1}{1 + \left(\dfrac{x}{3}\right)^2} \times \ldots$ where ... could be any value even 1

dM1: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 + \dfrac{1}{1 + \left(\dfrac{x}{3}\right)^2} \times \dfrac{1}{3}$

A1*: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3}{x^2 + 9}$

Alt (a) via compound angle identity they could pick up the first two marks

$$x = \frac{3\left(\tan y - \tan\dfrac{\pi}{6}\right)}{1 + \tan\dfrac{\pi}{6}\tan y} \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\left(1 + \tan\dfrac{\pi}{6}\tan y\right) \times 3\sec^2 y - 3\left(\tan y - \tan\dfrac{\pi}{6}\right) \times \tan\dfrac{\pi}{6}\sec^2 y}{\left(1 + \tan\dfrac{\pi}{6}\tan y\right)^2}$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(1 + \tan\dfrac{\pi}{6}\tan y\right)^2}{\left(1 + \tan\dfrac{\pi}{6}\tan y\right) \times 3\sec^2 y - 3\left(\tan y - \tan\dfrac{\pi}{6}\right) \times \tan\dfrac{\pi}{6}\sec^2 y}$$

Question Number	Scheme	Notes
8	$\int (2\cos x - \sin x)^2 \, dx = \int \left(4\cos^2 x - 4\sin x \cos x + \sin^2 x\right) dx$	M1
	$\int 4\sin x \cos x \, dx = \int 2\sin 2x \, dx = -\cos 2x$ or $\int 4\sin x \cos x \, dx = -2\cos^2 x \quad \text{or} \quad 2\sin^2 x$	M1
	$\int \left(4\cos^2 x + \sin^2 x\right) dx = \int \left(1 + 3\cos^2 x\right) dx = \int \left(1 + 3\left(\frac{\cos 2x + 1}{2}\right)\right) dx$ or $\int \left(4\cos^2 x + \sin^2 x\right) dx = \int \left(4\left(\frac{\cos 2x + 1}{2}\right) + \frac{1 - \cos 2x}{2}\right) dx$	M1
	$\int (2\cos x - \sin x)^2 \, dx = \frac{3}{4}\sin 2x + \cos 2x + \frac{5}{2}x (+c)$ or $\int (2\cos x - \sin x)^2 \, dx = \frac{3}{4}\sin 2x + 2\cos^2 x + \frac{5}{2}x (+c)$ or $\int (2\cos x - \sin x)^2 \, dx = \frac{3}{4}\sin 2x - 2\sin^2 x + \frac{5}{2}x (+c)$	A1A1
		(5)
		Total 5

Condone changes in variables throughout this solution as long as the answer is given in terms of x

M1: Expands to the form $p\cos^2 x + q\sin x\cos x + r\sin^2 x$

M1: Correct strategy for **integrating** $q\sin x\cos x$ (i.e. obtains $k\cos 2x$ or $k\sin^2 x$ or $k\cos^2 x$)

M1: Correct strategy for rewriting $p\cos^2 x + r\sin^2 x$ **into a form that can be integrated**.

Score for one of

- writes in terms of just $\sin^2 x$ and then uses $\sin^2 x = \frac{\pm 1 \pm \cos 2x}{2}$
- writes in terms of just $\cos^2 x$ and then uses $\cos^2 x = \frac{\pm 1 \pm \cos 2x}{2}$
- writes both $\sin^2 x$ and $\cos^2 x$ in terms of $\cos 2x$ with **at least one** of these via use of a correct/allowable form. That is $\sin^2 x = \frac{\pm 1 \pm \cos 2x}{2}$ or $\cos^2 x = \frac{\pm 1 \pm \cos 2x}{2}$

A1: Integrates and achieves 2 correct terms (of the 3 required terms)

NB: An unsimplified expression is acceptable for this mark so please check carefully. e.g. $2x + ... + \frac{1}{2}x$ counts as one correct term.

A1: Correct simplified integration (+ c not required).

An alternative solution is via "R cos" or "R sin" but the last two marks are unlikely to be achieved due to the fact that an exact answer is difficult to arrive at.

Question Number	Scheme	Notes
8	$\int (2\cos x - \sin x)^2 \, dx = \int 5\cos^2(x + 0.464) \, dx$	M1
	$= \int \frac{5\{\cos(2x + 0.927) + 1\}}{2} dx$	M1
	$\frac{5\sin(2x + 0.927)}{4} + \frac{5}{2}x$	M1
	$\frac{3\sin 2x}{4} + \cos 2x + \frac{5}{2}x + c$	A1A1
		(5)
		Total 5

M1: Writes $2\cos x - \sin x$ in the form $R\cos(x \pm \alpha)$ o.e. and squares.

Requires a full method so for the form $R\cos(x \pm \alpha)$

- requires $R^2 = 2^2 + 1^2$
- requires $\tan\alpha = \pm\frac{1}{2} \Rightarrow \alpha = ...$ in radians but condone in degrees

M1: Correct strategy for writing $\cos^2(x \pm \alpha)$ into a form that can be integrated using the double angle formula

$\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm 2\alpha)}{2}$ but condone $\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm \alpha)}{2}$

α should be in radians but condone in degrees

M1: Correct strategy for writing $\cos^2(x \pm \alpha)$ into a form that can be integrated using the double angle formula

$\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm 2\alpha)}{2}$ but condone $\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm \alpha)}{2}$

α should be in radians but condone in degrees

M1: It is for integrating $\cos(2x \pm \delta) \to \pm\sin(2x \pm \delta)$ following use of an acceptable double angle formula

A1: 2 correct terms of $\frac{3\sin 2x}{4} + \cos 2x + \frac{5}{2}x + c$

A1: Fully correct and simplified (+ c not required) or $\frac{5\sin(2x + 2\arctan 0.5)}{4} + \frac{5}{2}x$

Question Number	Scheme	Notes
8	$\int (2\cos x - \sin x)^2 \, dx = \int 5\sin^2(x - 1.107) \, dx$	M1
	$= \int \frac{5\{1 - \cos(2x - 2.21)\}}{2} dx$	M1
	$= \frac{5}{2}x - \frac{5\sin(2x - 2.21)}{4}$	M1
	$= \frac{5}{2}x + \frac{3\sin 2x}{4} + \cos 2x + c$	A1A1
		(5)
		Total 5