Question Number	Scheme	Marks
9(a)	$y=\sqrt{3+4\mathrm{e}^{x^2}}=\left(3+4\mathrm{e}^{x^2}\right)^{\frac{1}{2}}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(3+4\mathrm{e}^{x^2}\right)^{-\frac{1}{2}}\times 8x\mathrm{e}^{x^2}$	M1
	$=4x\mathrm{e}^{x^2}\left(3+4\mathrm{e}^{x^2}\right)^{-\frac{1}{2}}$	A1
		(2)
(b)	$\frac{\left(3+4\mathrm{e}^{x^2}\right)^{\frac{1}{2}}}{x}=4x\mathrm{e}^{x^2}\left(3+4\mathrm{e}^{x^2}\right)^{-\frac{1}{2}}$	M1
	$\frac{\left(3+4\mathrm{e}^{x^2}\right)}{x}=4x\mathrm{e}^{x^2}$	dM1
	$4x^2\mathrm{e}^{x^2}-4\mathrm{e}^{x^2}-3=0*$	A1*
		(3)
(c)	$\mathrm{f}(x)=4x^2\mathrm{e}^{x^2}-4\mathrm{e}^{x^2}-3\Rightarrow\mathrm{f}(1)=-3\quad\text{AND}\quad\mathrm{f}(2)=652\cdot\ldots$	M1
	Change of sign and f (x) is continuous hence root in (1, 2)	A1
		(2)
(d)	$4x^2\mathrm{e}^{x^2}-4\mathrm{e}^{x^2}-3=0\Rightarrow x^2=\frac{4\mathrm{e}^{x^2}+3}{4\mathrm{e}^{x^2}}=\frac{4+3\mathrm{e}^{-x^2}}{4}\Rightarrow x=\frac{1}{2}\sqrt{4+3\mathrm{e}^{-x^2}}$ *	B1*
		(1)
(e)(i)	$x_1=1\Rightarrow x_2=\frac{1}{2}\sqrt{4+3\mathrm{e}^{-1}}$	M1
	$x_3=1.0997$	A1
(ii)	$\alpha=1.1051$	A1
		(3)
		Total 11

(a)

M1: Differentiates using the chain rule to obtain $kx\mathrm{e}^{x^2}\left(3+4\mathrm{e}^{x^2}\right)^{-\frac{1}{2}}$ OR $k\mathrm{e}^{x^2}\left(3+4\mathrm{e}^{x^2}\right)^{-\frac{1}{2}}$

If they are using implicit differentiation accept $\frac{kx\mathrm{e}^{x^2}}{y}$ or $\frac{k\mathrm{e}^{x^2}}{y}$

A1: Correct derivative in simplest form. Also award for $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4x\mathrm{e}^{x^2}}{y}$ o.e. simplified answer

(b)

M1: Sets their $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(3+4\mathrm{e}^{x^2}\right)^{\frac{1}{2}}}{x}$ or equivalent such as use of $y=mx$ with $m=$ their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $y=\left(3+4\mathrm{e}^{x^2}\right)^{\frac{1}{2}}$.

Allow with another variable $x\to\alpha$

dM1: Multiplies up to eliminate the square root. Allow a consistent use of another variable $x\to\alpha$

Dependent upon the previous M1 and a suitable $\frac{\mathrm{d}y}{\mathrm{d}x}$, that is one of the form that scored an M1 in part (a)

A1*: Correct proof

An alternative method may be seen using the equation of the line with gradient $\frac{dy}{dx}$ and point $\left(\alpha, \sqrt{3+4e^{\alpha^2}}\right)$

M1: For $y-\sqrt{3+4e^{\alpha^2}} = "\left.\frac{dy}{dx}\right|_{x=\alpha}"(x-\alpha)$ and sets $(x,y)=(0,0)$

dM1: Proceeds as the main method with the same condition on the gradient

A1*: Correct proof. Condone with variable α

(c)

M1: Attempts both f (1) and f (2) (or tighter) and obtains at least one value correct to 1sf rounded or truncated.

Condone f (2) = $12e^4 - 3$ If f or *y* is not stated assume that f (*x*) = $4x^2e^{x^2} - 4e^{x^2} - 3$

A1: Requires

- both values either correct or rounded/ truncated with accuracy to at least 1 sf
- a reference to a sign change e.g. $f(1) = -3 < 0$, $f(2) = 652 > 0$ or f(1) × f(2) < 0
- a mention of continuity
- a minimal conclusion, e.g. hence root

(d)

B1*: Correct proof.

Look for the following evidence

- the line $4x^2e^{x^2} - 4e^{x^2} - 3 = 0$ but may be implied by $4x^2e^{x^2} = 4e^{x^2} + 3$ o.e.
- the line $x^2 = \frac{4e^{x^2}+3}{4e^{x^2}}$ o.e $x^2 = 1 + \frac{3}{4e^{x^2}}$ or $4x^2 = \frac{4e^{x^2}+3}{e^{x^2}}$ o.e $4x^2 = 4 + \frac{3}{e^{x^2}}$
- correct square root work leading to the given answer

(e)

M1: Attempts to use the iteration formula.

This may be implied by awrt 1.13 for x_2 or awrt 1.10 for x_3 or sight of a correctly embedded value.

It cannot be awarded for just the value of α

A1: awrt 1.0997

A1: 1.1051 cao following the award M1

Question Number	Scheme	Marks
10(a)	$\left(F=\right)35$	B1
		(1)
(b)	$200=\dfrac{350\mathrm{e}^{15k}}{9+\mathrm{e}^{15k}}\Rightarrow 1800+200\mathrm{e}^{15k}=350\mathrm{e}^{15k}\Rightarrow 150\mathrm{e}^{15k}=1800$	M1
	$\mathrm{e}^{15k}=\dfrac{1800}{150}\Rightarrow 15k=\ln 12\Rightarrow k=\dfrac{1}{15}\ln 12*$	dM1A1*
		(3)
(c)	$F=\dfrac{350\mathrm{e}^{kt}}{9+\mathrm{e}^{kt}}\Rightarrow \dfrac{\mathrm{d}F}{\mathrm{d}t}=\dfrac{350k\mathrm{e}^{kt}\left(9+\mathrm{e}^{kt}\right)-350\mathrm{e}^{kt}\left(k\mathrm{e}^{kt}\right)}{\left(9+\mathrm{e}^{kt}\right)^2}$	M1A1
	$\dfrac{3150k\mathrm{e}^{kt}}{\left(9+\mathrm{e}^{kt}\right)^2}=10\Rightarrow 315k\mathrm{e}^{kt}=81+18\mathrm{e}^{kt}+\mathrm{e}^{2kt}\Rightarrow \mathrm{e}^{2kt}+\left(18-315k\right)\mathrm{e}^{kt}+81=0$	M1
	$\mathrm{e}^{2kt}+\left(18-315k\right)\mathrm{e}^{kt}+81=0\Rightarrow \mathrm{e}^{kt}=\dfrac{315k-18\pm\sqrt{\left(18-315k\right)^2-4\times 81}}{2}\Rightarrow kt=...$	M1
	$T=$ awrt 5.7, 20.8	A1
		(5)
		Total 9

(a)
B1: Correct value, 35
(b)
M1: Uses $F = 200$ and $t = 15$ and reaches $A\mathrm{e}^{15k} = B$ where $A\times B>0$
dM1: Proceeds using correct order of operations to obtain a value for k
Look for $A\mathrm{e}^{15k}=B\Rightarrow \mathrm{e}^{15k}=\dfrac{B}{A}\Rightarrow 15k=\ln\dfrac{B}{A}\Rightarrow k=...$
or Alternatively $A\mathrm{e}^{15k}=B\Rightarrow \ln A+15k=\ln B\Rightarrow 15k=\ln B-\ln A\Rightarrow k=...$
A1*: Correct proof with all necessary steps shown.
Minimum acceptable solution $200=\dfrac{350\mathrm{e}^{15k}}{9+\mathrm{e}^{15k}}\Rightarrow 150\mathrm{e}^{15k}=1800\Rightarrow \mathrm{e}^{15k}=12\Rightarrow k=\dfrac{1}{15}\ln 12$
(c) Allow the whole of part (c) to be done with the letter k, the exact value for k or using k = awrt 0.166
M1: Correct attempt of the quotient (product or chain) rule.
For the Quotient Rule look for $\dfrac{\mathrm{d}F}{\mathrm{d}t}=\dfrac{P\mathrm{e}^{kt}\left(9+\mathrm{e}^{kt}\right)-Q\mathrm{e}^{kt}\left(\mathrm{e}^{kt}\right)}{\left(9+\mathrm{e}^{kt}\right)^2}\qquad P,Q>0$
For the Product Rule look for $\dfrac{\mathrm{d}F}{\mathrm{d}t}=P\mathrm{e}^{kt}\left(9+\mathrm{e}^{kt}\right)^{-1}\pm Q\mathrm{e}^{kt}\mathrm{e}^{kt}\left(9+\mathrm{e}^{kt}\right)^{-2}\qquad P,Q>0$
For Chain Rule look for $F=A\pm\dfrac{B}{9+e^{kt}}\Rightarrow\dfrac{\mathrm{d}F}{\mathrm{d}t}=Q\mathrm{e}^{kt}\left(9+\mathrm{e}^{kt}\right)^{-2}$ Q is a constant
A1: Correct differentiation, which may be unsimplified.