A1: cso $x=\dfrac{-3+\sqrt{105}}{4}$ or awrt 1.81 ONLY. If $x=\dfrac{-3-\sqrt{105}}{4}$ or awrt $-3.31$ is also written down it must be rejected. ISW after a correct answer. There must be evidence of dM1 to award this mark

| Question Number | Scheme | Marks |
|---|---|---|
| **6. (a)** | $\mathrm{f}(x)=\dfrac{5x-3}{x-4}\Rightarrow \mathrm{f}'(x)=\dfrac{5(x-4)-(5x-3)}{(x-4)^2}=\dfrac{k}{(x-4)^2}$ | M1 dM1 |
| | States that $\mathrm{f}'(x)=\dfrac{-17}{(x-4)^2}\Rightarrow \mathrm{f}'(x)<0$ hence decreasing * cso | A1* |
| | | **(3)** |
| **(b)** | $y=\dfrac{5x-3}{x-4}\Rightarrow xy-4y=5x-3\Rightarrow xy-5x=4y-3$ | M1 |
| | $\Rightarrow x=\dfrac{4y-3}{y-5}$ So $\mathrm{f}^{-1}(x)=\dfrac{4x-3}{x-5}$ | A1 |
| | Domain $x>5$ | B1 |
| | | **(3)** |
| **(c) (i)** | $\mathrm{ff}(x)=\dfrac{5\times\dfrac{5x-3}{x-4}-3}{\dfrac{5x-3}{x-4}-4}$ | M1 |
| | $\mathrm{ff}(x)=\dfrac{5\times(5x-3)-3(x-4)}{5x-3-4(x-4)}=\dfrac{22x-3}{x+13}$ | dM1 A1 |
| **(ii)** | $5<\mathrm{ff}(x)<22$ | B1, B1 |
| | | **(5)** |
| | | **(11 marks)** |

(a)

M1: Attempts to use the quotient rule to achieve a form $\dfrac{p(x-4)-q(5x-3)}{(x-4)^2}$ with $p,q>0$ OR

attempts the product rule to achieve a form $p(x-4)^{-1}\pm(5x-3)(x-4)^{-2}$

Condone attempts in which terms such as $(x-4)^2$ may have been multiplied out incorrectly

dM1: Proceeds to $\mathrm{f}'(x)=\dfrac{k}{(x-4)^2}$ Allow the lhs to be $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

A1*: Requires a correct $\mathrm{f}'(x)$, a correct statement such as "$\mathrm{f}'(x)<0$ " o.e (such as $\mathrm{f}'(x)$ is negative ) and a minimal conclusion which could be ✓ or QED or □

It cannot be awarded from substituting in single values of *x*. It cannot be based on incorrect mathematics.

"There is a minus sign" or "$-17<0$ " is insufficient without the further statement $\mathrm{f}'(x)<0$ (o.e) followed by a minimal conclusion.

(b)

M1 Attempts to change the subject. Look for cross multiplication with an attempt to collect terms with *x*'s (or replaced *y*'s) on one side of the equation and non *x* terms on the other side

A1 $\mathrm{f}^{-1}(x)=\dfrac{4x-3}{x-5}$ o.e Condone the lhs as $\mathrm{f}^{-1}$ and even *y*. The notation $\mathrm{f}^{-1}:x\mapsto\dfrac{4x-3}{x-5}$ is fine

B1 Correct domain $x > 5$

(c) (i)

M1 Attempts to substitute $\frac{5x-3}{x-4}$ into f. Condone minor slips but the form of the expression must be correct

dM1 Multiplies all terms on numerator and denominator by $x-4$. Condone missing brackets

Alternatively writes both the numerator and denominator as single fractions over $(x-4)$

A1 $(\text{ff}(x)) = \frac{22x-3}{x+13}$ Condone a missing left hand side or being set as $y =$

(c)(ii)

B1: Achieves the value of one "end". It is not just for the number so f f < 5 is B0

Condone non strict inequalities here so $5 \leqslant \text{ff} < 21$ would be fine for B1 B0

B1: Fully correct. Accept $5 < \text{ff} < 22$, $(5, 22)$. Allow with $y$ or range but $5 < x < 22$ would be B1 B0

Methods where candidates "split up" fraction $\frac{5x-3}{x-4} = 5 + \frac{17}{x-4}$

(a)

M1:Attempts to split into form $5 + \frac{17}{x-4}$ AND attempts to use the chain rule.

Look for $\frac{5x-3}{x-4} \to A + \frac{B}{x-4}$ with a differential of $\frac{k}{(x-4)^2}$

dM1: Proceeds to $\text{f}'(x) = \frac{k}{(x-4)^2}$ Allow the lhs to be $\frac{\text{d}y}{\text{d}x}$

A1*: $\text{f}'(x) = \frac{-17}{(x-4)^2}$ and states $\text{f}'(x) < 0$ (for all $x$) so f is decreasing function

(b)

M1: Attempts to split $y = \frac{5x}{x+3}$ into form $y = 5 + \frac{17}{x-4}$ and then attempts to make $x$ the subject.

Look for $y = \frac{5x-3}{x-4} = A + \frac{B}{x-4}$ with progress to a form $x - 4 = \frac{B}{\pm y \pm A}$

A1: $\text{f}^{-1}(x) = \frac{17}{x-5} + 4$ or equivalent

B1: $x > 5$

(c)

M1: Attempts to split $y = \frac{5x}{x+3}$ into form $y = 5 + \frac{17}{x-4}$ and then attempts $\text{ff}(x) = 5 + \frac{17}{5 + \frac{17}{x-4} - 4}$

dM1: Uses a correct method to combine into a single fraction of the required form

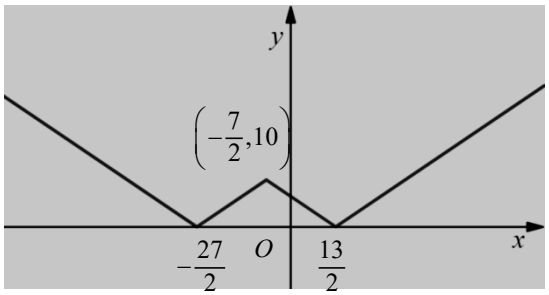
A1: $\text{ff}(x) = \frac{22x-3}{x+13}$

(c)(ii)

B1: Achieves the value of one "end". It is not just for the number so ff < 5 is B0

Condone non strict inequalities here so $5 \leqslant \text{ff} < 21$ would be fine for B1 B0

B1: Fully correct. Accept $5 < \text{ff} < 22$, $(5,22)$. Allow with $y$ or range but $5 < x < 22$ would be B1 B0

| Question Number | Scheme | Marks |
|---|---|---|
| **7 (a)** | $\left(-\frac{7}{2}, -10\right)$ | B1 B1 **(2)** |
| **(b)** | Attempts to solve either $\frac{1}{2}(2x+7)-10...\frac{1}{3}x+1$ Or $-\frac{1}{2}(2x+7)-10...\frac{1}{3}x+1$ | M1 |
| | Both correct critical values $x...-\frac{87}{8}, \frac{45}{4}$ | A1 |
| | Selects outside region for their critical values | dM1 |
| | Correct solution $x \leqslant -\frac{87}{8}, x \geqslant \frac{45}{4}$ | A1 **(4)** |
| **(c)** |   Shape | B1 |
| | Maximum | B1 ft |
| | One correct minimum | B1 |
| | Fully correct | B1 **(4)** **(10 marks)** |

(a) Must be answered in (a). We cannot imply from work in (c)

B1: One correct coordinate. Allow as $x = -\frac{7}{2}, y = -10$ . Allow exact equivalents , e.g. $x = -\frac{14}{4}$

B1: Both coordinates correct. Allow as $x = -\frac{7}{2}, y = -10$. Allow exact equivalents , e.g. $x = -\frac{14}{4}$

(b)

M1: Attempts to solve a correct equation or inequality. They must proceed as far as $x$ ...

Condone slips where they attempt to change the subject in an attempt to make $|2x+7|$ the subject.

So either of $\frac{1}{2}|2x+7|-10...\frac{1}{3}x+1 \Rightarrow |2x+7| = ax+b \Rightarrow 2x+7 = ax+b$

or $\frac{1}{2}|2x+7|-10...\frac{1}{3}x+1 \Rightarrow |2x+7| = ax+b \Rightarrow -2x-7 = ax+b$ are condoned for the M1

A1: Correct critical values $x...-\frac{87}{8}, \frac{45}{4}$ which may be part of an incorrect inequality

dM1: Selects outside region for their critical values. Allow these to be written as one inequality. Allow the strict inequalities here. It is dependent upon having attempted to solve **one** correct equation