Question Number	Scheme	Marks
9 (i)	$2\sec^2 x - 3\tan x = 2$	
	$2\left(1+\tan^2 x\right) - 3\tan x = 2$	M1
	$2\tan^2 x = 3\tan x \Rightarrow \tan x = \frac{3}{2}, \left(\tan x = 0\right)$	dM1
	$x =$ awrt $0.983, \pi$	A1, B1
		(4)
(ii)	$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} \equiv \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta} \equiv \frac{\sin\left(3\theta-\theta\right)}{\sin\theta\cos\theta}$	M1
	$\equiv \frac{\sin\left(3\theta-\theta\right)}{\frac{1}{2}\sin 2\theta}$	dM1, A1
	$\equiv \frac{\cancel{\sin 2\theta}}{\frac{1}{2}\cancel{\sin 2\theta}} \equiv 2$ *	A1*
		(4)
		(8 marks)

(i)

M1: Attempts to use $\sec^2 x = \pm 1 \pm \tan^2 x$ within the given equation, condoning slips

There are alternatives using for example $\sec^2 x = \frac{1}{\cos^2 x}$, $\tan x = \frac{\sin x}{\cos x}$ and $\cos^2 x = 1 - \sin^2 x$

FYI the correct equation using this method would be $2\sin^2 x - 3\sin x\cos x = 0$

dM1: Valid method of solving a quadratic equation in $\tan x$. Condone division by $\tan x$

In the alternative the method of solving is similar $\sin x\left(2\sin x - 3\cos x\right) = 0 \Rightarrow \tan x = \frac{3}{2}$

A1: $x =$ awrt 0.983 as the only solution to $\tan x = \frac{3}{2}$ in the region $\left(0, \pi\right]$

B1: $x = \pi$ or awrt 3.14 (as the only solution in the range for $\tan x = 0$ or $\sin x = 0$ in the region $\left(0, \pi\right]$).

Condone the inclusion of 0 as that is outside the range

Condone 180° if both angles have been given in degrees. Ignore any solution outside the range

(ii)

M1: Attempts to form a single fraction and uses

- either the compound angle formula on the numerator. Allow $\sin\left(3\theta \pm \theta\right)$
- or the double angle on the denominator. Allow $\sin\theta\cos\theta = k\sin 2\theta$

dM1: Attempts to form a single fraction and uses both

- the compound angle formula on the numerator. Must be $\sin\left(3\theta - \theta\right)$
- and the double angle on the numerator or denominator. Allow $\sin\theta\cos\theta = k\sin 2\theta$

A1: For reaching the form $\frac{\sin\left(3\theta-\theta\right)}{\frac{1}{2}\sin 2\theta}$ **or** $\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

A1*: Fully complete proof.

Alt soln

M1: Attempts compound angle formulae for both $\sin 3\theta$ and $\cos 3\theta$. Condone **only** sign slips

$$\frac{\sin 3\theta}{\sin\theta}-\frac{\cos 3\theta}{\cos\theta}=\frac{\sin(2\theta+\theta)}{\sin\theta}-\frac{\cos(2\theta+\theta)}{\cos\theta}=\frac{\sin 2\theta\cos\theta+\cos 2\theta\sin\theta}{\sin\theta}-\frac{\cos 2\theta\cos\theta-\sin 2\theta\sin\theta}{\cos\theta}$$

dM1: This mark is dependent upon the previous one having been awarded. It can be awarded for one of

- attempting to use the double angle formula for $\sin 2\theta$ (condone $\sin 2\theta = k\sin\theta\cos\theta$ if used consistently), then dividing and cancelling out the terms in $\cos 2\theta$ to produce an expression in just $a\cos^2\theta \pm b\sin^2\theta$

  E.g. $\frac{2\cancel{\sin\theta}\cos\theta\cos\theta}{\cancel{\sin\theta}}+\frac{\cos 2\theta\cancel{\sin\theta}}{\cancel{\sin\theta}}-\frac{\cos 2\theta\cancel{\cos\theta}}{\cancel{\cos\theta}}\pm\frac{2\sin\theta\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}=....$

- as above but may use $\cos 2\theta = \pm 1\pm 2\sin^2\theta$ o.e. to produce an expression in just $a\cos^2\theta + b\sin^2\theta + c$

  E.g.

$$\frac{2\cancel{\sin\theta}\cos\theta\cos\theta}{\cancel{\sin\theta}}+\frac{\left(\cos^2\theta-\sin^2\theta\right)\cancel{\sin\theta}}{\cancel{\sin\theta}}-\frac{\left(\cos^2\theta-\sin^2\theta\right)\cancel{\cos\theta}}{\cancel{\cos\theta}}\pm\frac{2\sin\theta\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}=...$$

- Uses appropriate expressions for $\sin 2\theta$ and $\cos 2\theta$ then uses a correct common factor to produce a single fraction in just $\sin\theta$ and $\cos\theta$

E.g.

$$\frac{2\sin\theta\cos\theta\cos^2\theta+\left(\cos^2\theta-\sin^2\theta\right)\sin\theta\cos\theta}{\sin\theta\cos\theta}-\frac{\left(\cos^2\theta-\sin^2\theta\right)\sin\theta\cos\theta\pm 2\sin\theta\cos\theta\sin^2\theta}{\sin\theta\cos\theta}=...$$

A1: For a correct intermediate line in single angles that leads to the given answer

In most methods it is for one of

- $\frac{2\cancel{\sin\theta}\cos\theta\cos\theta}{\cancel{\sin\theta}}+\frac{2\sin\theta\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}$ or equivalent
- $2\cos^2\theta+1-2\sin^2\theta+1-2\cos^2\theta+2\sin^2\theta$
- $\frac{2\sin\theta\cos^3\theta+2\sin^3\theta\cos\theta}{\sin\theta\cos\theta}$
- $\frac{2\cancel{\sin\theta}\cos\theta\cos\theta+\cos 2\theta\cancel{\sin\theta}}{\cancel{\sin\theta}}-\frac{\cos 2\theta\cancel{\cos\theta}-2\sin\theta\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}}$ or equivalent.

We can see clearly that the $\cos 2\theta$ terms cancel so that it is effectively an expression in single angles

A1*: Fully complete proof with no errors. You may tolerate the odd notational slip.

Withhold this mark if there are obvious and repeated notational errors

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Most answers seen will be a combination of these.

Generally the marks are awarded for

M1: Makes a positive step towards achieving the given answer

dM1: Makes all the correct steps towards the proof but allow slips

A1: A correct line that usually only requires the use of the pythagorean identity to reach the given answer

If you see the triple angle identities used (and it is incorrect) please send to review

$$\frac{\sin 3\theta}{\sin\theta}-\frac{\cos 3\theta}{\cos\theta}\equiv\frac{3\sin\theta-4\sin^3\theta}{\sin\theta}-\frac{4\cos^3\theta-3\cos\theta}{\cos\theta}\equiv\left(3-4\sin^2\theta\right)-\left(4\cos^2\theta-3\right)\equiv 6-4\left(\sin^2\theta+\cos^2\theta\right)\equiv 2$$

Question Number	Scheme	Marks
10 (a)	$x = y\mathrm{e}^{2y} \Rightarrow \quad \dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y} + 2y\mathrm{e}^{2y}$	M1, A1
	$\Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{x}{y} + 2x$	
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{\frac{x}{y} + 2x}$	dM1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y}{x + 2xy} = \dfrac{y}{x(1+2y)}$ *	A1*
		(4)
(b)	Deduces $y = -\dfrac{1}{2}$	B1
	Substitutes $y = -\dfrac{1}{2} \Rightarrow x = -\dfrac{1}{2\mathrm{e}}$ Range for k $-\dfrac{1}{2\mathrm{e}} < k < 0$	M1, A1
		(3)
		(7 marks)

(a)

M1: For attempting to differentiate with respect to y.

Uses the product rule on $y\mathrm{e}^{2y} \Rightarrow \quad \mathrm{e}^{2y} + \ldots y\mathrm{e}^{2y}$. The left hand side may be missing/incorrect

A1: Correct differentiation E.g. $\dfrac{\mathrm{d}x}{\mathrm{d}y} = \mathrm{e}^{2y} + 2y\mathrm{e}^{2y}$

dM1: Full method to get $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of just x and y.

This requires, in any order

- a correct attempt to invert, e.g. not inverting each term in a sum of terms
- e^{2y} being fully replaced. You should see e^{2y} being replaced by $\dfrac{x}{y}$ or equivalent and $y\mathrm{e}^{2y}$ being replaced by x

A1*: Correct proof. All relevant steps should be shown and there should be no errors.

If you feel that it hasn't been fully shown then please award M1 A1 dM1 A0

(b) Now being marked B1 M1 A1. On epen it is set up M1 A1 A1

B1: For deducing left hand end occurs when $y = -\dfrac{1}{2}$.

M1: For attempting to find x when $y = -\dfrac{1}{2} \Rightarrow x = \ldots$ This may be implied by k = awrt -0.183 or -0.184

A1: $-\dfrac{1}{2\mathrm{e}} < k < 0$ or **exact** equivalent such as e.g. $-\dfrac{1}{2}\mathrm{e}^{-1} < k < 0$, $k > -\dfrac{1}{2}\mathrm{e}^{-1}$ **and** $k < 0$,

$\left\{k : k > -\dfrac{1}{2\mathrm{e}}\right\} \cap \{k : k < 0\}$

This must be correct so the candidate cannot have two separate inequalities with an "**or**" between

This must be the range for k not x