Question Number	Scheme	Marks
8(a)	200	B1
		(1)
(b)	600	B1
		(1)
(c)	$500 = \dfrac{600\mathrm{e}^{0.3t}}{2+\mathrm{e}^{0.3t}} \Rightarrow 100\mathrm{e}^{0.3t} = 1000, \Rightarrow \mathrm{e}^{0.3t} = 10$	M1, A1
	$\Rightarrow t = \dfrac{\ln 10}{0.3} = 7$ years 8 months	dM1 A1
		(4)
(d)	$N = \dfrac{600\mathrm{e}^{0.3t}}{2+\mathrm{e}^{0.3t}} \Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{\left(2+\mathrm{e}^{0.3t}\right)\times 180\mathrm{e}^{0.3t} - 180\mathrm{e}^{0.3t}\times\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$	M1 A1
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{360\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$	A1 **(3)**
(e)	$8 = \dfrac{360\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2} \Rightarrow 8\left(\mathrm{e}^{0.3t}\right)^2 - 328\left(\mathrm{e}^{0.3t}\right) + 32 = 0$	M1
	$\mathrm{e}^{0.3t} = \dfrac{41+\sqrt{1665}}{2}$	dM1
	$(t=)\dfrac{\ln\left(\dfrac{41+\sqrt{1665}}{2}\right)}{0.3}$	ddM1
	$(T=)$awrt 12.4	A1
		(4)
		(13 marks)

(a)

B1: 200

(b)

B1: 600

(c)

M1: Sets $N = 500$ and proceeds to $C\mathrm{e}^{0.3t} = D$ or equivalent

A1: $\mathrm{e}^{0.3t} = 10$. If ln's are taken earlier it would be for $\ln 100 + 0.3t = \ln 1000$

dM1: Full method to find a value for *t* using correct log work.

A1: 7 years 8 months. Accept 7 years 9 months following a correct value of *t*.

(d)

M1: Uses the quotient rule to obtain an expression of the form $\dfrac{\left(2+\mathrm{e}^{0.3t}\right)\times a\mathrm{e}^{0.3t}-b\mathrm{e}^{0.3t}\times\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$

In general condone missing brackets for the M mark.

A correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $\dfrac{vu'-uv'}{v^2}$ etc.

E.g. if they quote $u = 600\mathrm{e}^{0.3t}$ and $v = 2 + \mathrm{e}^{0.3t}$ and do not make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have *v* rather than v^2 in the denominator.

A1: Correct derivative in any form e.g. $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{\left(2+\mathrm{e}^{0.3t}\right)\times180\mathrm{e}^{0.3t}-180\mathrm{e}^{0.3t}\times\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$

A1: Correctly obtains $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{360\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$

Withhold this mark if you see $\mathrm{e}^{0.3t}\times\mathrm{e}^{0.3t}$ written as $\mathrm{e}^{0.3t^2}$ or $\mathrm{e}^{0.09t}$

.....

Alt (d)

M1 A1 : $N=\dfrac{600\mathrm{e}^{0.3t}}{2+\mathrm{e}^{0.3t}}\Rightarrow N=600-\dfrac{1200}{2+\mathrm{e}^{0.3t}}\Rightarrow\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{1200\times0.3\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$

Score M1 for splitting $\dfrac{600\mathrm{e}^{0.3t}}{2+\mathrm{e}^{0.3t}}$ into $A\pm\dfrac{B}{2+\mathrm{e}^{0.3t}}$ leading to $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{k\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$

and A1 for correct derivative in any form.

A1: $\dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{360\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$ (following fully correct work)

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May also see product rule in (d):

$$N=\frac{600\mathrm{e}^{0.3t}}{2+\mathrm{e}^{0.3t}}=600\mathrm{e}^{0.3t}\left(2+\mathrm{e}^{0.3t}\right)^{-1}\Rightarrow\frac{\mathrm{d}N}{\mathrm{d}t}=180\mathrm{e}^{0.3t}\left(2+\mathrm{e}^{0.3t}\right)^{-1}-180\mathrm{e}^{0.3t}\times\mathrm{e}^{0.3t}\left(2+\mathrm{e}^{0.3t}\right)^{-2}$$

$$=\frac{180\mathrm{e}^{0.3t}}{2+\mathrm{e}^{0.3t}}-\frac{180\mathrm{e}^{0.6t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}=\frac{360\mathrm{e}^{0.3t}+180\mathrm{e}^{0.6t}-180\mathrm{e}^{0.6t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}=\frac{360\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$$

Score M1 for an expression of the form $a\mathrm{e}^{0.3t}\left(2+\mathrm{e}^{0.3t}\right)^{-1}-b\mathrm{e}^{0.3t}\times\mathrm{e}^{0.3t}\left(2+\mathrm{e}^{0.3t}\right)^{-2}$ and A1 A1 as above.

A correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $vu'+uv'$ etc.

(e)

M1: Sets $\dfrac{\mathrm{d}N}{\mathrm{d}t}=8$ and proceeds to a quadratic in $\mathrm{e}^{0.3t}$

dM1: Correct attempt to solve the quadratic in $\mathrm{e}^{0.3t}$. Condone both roots to be found

ddM1: Correct attempt to find the value of *t* for $\mathrm{e}^{0.3t}=k$ where $k>0$ using correct log work

A1: Achieves $(T=)$awrt 12.4

Question Number	Scheme	Marks
9(a)	$R = 13$	B1
	$\tan\alpha = \frac{5}{12} \Rightarrow \alpha = \text{awrt } 0.395$	M1A1
		(3)
(b)	$g(\theta) = 10 + 13\sin\left(2\theta - \frac{\pi}{6} - 0.395\right)$	
(i)	(i) Minimum value is -3	B1 ft
(ii)	$2\theta - \frac{\pi}{6} - 0.395 = \frac{3\pi}{2} \Rightarrow \theta = \text{awrt } 2.82$	M1 A1
		(3)
(c)	$h(\beta) = 10 - 169\sin^2(\beta - 0.395)$	
	$-159 \leqslant h \leqslant 10$	M1 A1
		(2)
		(8 marks)

(a)

B1: $R = 13$ ($R = \pm 13$ is B0)

M1: $\tan\alpha = \pm\frac{5}{12}$, $\tan\alpha = \pm\frac{12}{5} \Rightarrow \alpha = ...$

If R is used to find α accept $\sin\alpha = \pm\frac{5}{R}$ or $\cos\alpha = \pm\frac{12}{R} \Rightarrow \alpha = ...$

A1: $\alpha = \text{awrt } 0.395$ Note that the degree equivalent $\alpha = \text{awrt } 22.6°$ is A0

(b)(i)

B1ft: States the value of $10 - R$ following through their R.

(b)(ii)

M1: Attempts to solve $2\theta - \frac{\pi}{6} \pm "0.395" = \frac{3\pi}{2} \Rightarrow \theta = ...$

A1: $\theta = \text{awrt } 2.82$. No other values should be given

(c)

M1: Achieves one of the end values, either -159 (or 10 – (their R)^2 evaluated) or 10

A1: Fully correct range $-159 \leqslant h \leqslant 10$, $-159 \leqslant h(\beta) \leqslant 10$, $-159 \leqslant \text{range} \leqslant 10$, $-159 \leqslant h(x) \leqslant 10$, $[-159, 10]$ or equivalent correct ranges.