Question Number	Scheme	Marks
4 (a)	$\mathrm{ff}(6)=\mathrm{f}(13)=-1$	M1 A1 **(2)**
(b)	Attempts $21+2(2-x)=5x\Rightarrow x=...$ or $21-2(x-2)=5x\Rightarrow x=...$ $x=\frac{25}{7}$ only	M1 A1 **(2)**
(c)	Either $k<21$ or $k\geqslant 17$ $17\leqslant k<21$	M1 A1 **(2)**
(d)	$a=\frac{1}{7}$ $b=4$	B1 B1 **(2)** **(8 marks)**

(a)

M1 For attempting f "twice". Award for sight of $\mathrm{f}(13)$ or $21-2\left|2-\left(21-2\left|2-x\right|\right)\right|$ followed by a value.

A1 For -1

(b)

M1 Attempts $21+2(2-x)=5x\Rightarrow x=...$

A1 For $x=\frac{25}{7}$ only. Do not isw if students go on to find additional solutions unless they identify this as the only answer by rejecting others.

(c)

M1 For either $k<21$ or $k\geqslant 17$ Condone for this mark $k\leqslant 21$ or $k>17$

Alt: Allow for **both** $k = 17$ and $k = 21$ identified as critical values even if no inequalities are given.

A1 $17\leqslant k<21$ May be written as separate inequalities. Accept alternative notations for the range such as $\left[17,21\right)$

Do not accept $17\leqslant \mathrm{f}(x)<21$

(d)

B1 Either $a=\frac{1}{7}$ **or** $b=4$

B1 Both $a=\frac{1}{7}$ **and** $b=4$ Allow both marks if $y=\frac{1}{7}\mathrm{f}(x-4)$ is given.

Question Number	Scheme	Marks
5 (a)	$\sin 3x \equiv \sin(2x+x) \equiv \sin 2x\cos x + \cos 2x \sin x$	M1
	$\equiv 2\sin x\cos x\cos x + \left(1-2\sin^2 x\right)\sin x$	M1
	$\equiv 2\sin x\left(1-\sin^2 x\right) + \left(1-2\sin^2 x\right)\sin x$	ddM1
	$\equiv 3\sin x - 4\sin^3 x$ *	A1*
		(4)
(b)	$\int_0^{\frac{\pi}{3}} \sin^3 x \, \mathrm{d}x = \int_0^{\frac{\pi}{3}} \frac{3}{4}\sin x - \frac{1}{4}\sin 3x \, \mathrm{d}x$	M1
	$= \left[-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x\right]_0^{\frac{\pi}{3}}$	dM1 A1
	$= \frac{5}{24}$	A1
		(4)
		(8 marks)

(a)

M1 Uses $\sin 3x = \sin(2x+x) = \pm\sin 2x\cos x \pm \cos 2x\sin x$

M1 Uses the correct identity for $\sin 2x = 2\sin x\cos x$ and any correct identity for $\cos 2x$

ddM1 Dependent upon both previous M's. It is for using $\cos^2 x = 1-\sin^2 x$ to get an equation in only $\sin x$

A1* Fully correct solution with correct notation within their proof. Examples of incorrect notation include use of $\sin x^2$ instead of $\sin^2 x$ or use of sin instead of sin *x* and so on. Penalise in the A mark only for such.

Note: The ddM mark and final A mark may be score by substituting $\sin^2 x = 1-\cos^2 x$ into the right hand side of the equation to reach an identical expression to an expanded left hand side ("working from both sides"), with the A mark then awarded for correct work leading to identical expressions **and** an minimal conclusion given (e.g. //)

Note You may see use of De Moivre's Theorem from an FP3 candidate. This can score full credit if carried out correctly. If there are errors or you are unsure then send to review.

If attempted in reverse:

M1 $3\sin x - 4\sin^3 x = 3\sin x - 2\sin^2 x\sin x - 2\sin x(1-\cos^2 x) = \sin x - 2\sin x\frac{1}{2}(1-\cos 2x) + \sin 2x\cos x$

Uses $\sin^3 x = \sin x \times \sin^2 x$ with either $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ OR $\sin^2 x = 1-\cos^2 x$ and $2\sin x\cos x = \sin 2x$

M1 Uses both steps above to get to an equation with sin2*x* and cos*2x*

ddM1 Gathers terms to reach $= \cos 2x\sin x + \sin 2x\cos x$

A1 Completes the proof uses $\cos 2x\sin x + \sin 2x\cos x = \sin 3x$ with no errors seen.

(b)

M1 Attempts to use part (a) to simplify. Accept $\int \sin^3 x \, \mathrm{d}x = \int A\sin x + B\sin 3x \, \mathrm{d}x$

dM1 $\int A\sin x + B\sin 3x \, \mathrm{d}x \to a\cos x + b\cos 3x$

A1 $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$ oe (not a multiple of this (unless recovered))

A1 CSO $= \frac{5}{24}$

Note an answer of A1 $\frac{5}{24}$ with no supporting working scores no marks as algebraic integration is specified. But alternative methods of integration are permissible. Two alternatives are:

Question Number	Scheme	Marks
5 (b) Alt 1	$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \int_0^{\frac{\pi}{3}} \sin x\left(1-\cos^2 x\right) dx = \int_0^{\frac{\pi}{3}} \sin x - \sin x \cos^2 x \, dx$	M1
	$= \left[-\cos x - \left(-\frac{\cos^3 x}{3}\right)\right]_0^{\frac{\pi}{3}}$	dM1A1
	$= \frac{5}{24}$	A1
		(4)
(b) Alt 2	$u = \cos x \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow \int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \int_1^{\frac{1}{2}} u^2 - 1 \, du$	M1
	$= \left[\frac{u^3}{3} - u\right]_1^{\frac{1}{2}}$	dM1 A1
	$= \frac{5}{24}$	A1
		(4)
		(8 marks)

Notes: First three marks as follows (final A is as main scheme).

Alt 1

M1: Splits as $\sin^3 x = \sin x \sin^2 x$ and applies $\sin^2 x = 1 - \cos^2 x$ to get the integrand into and integrable form.

dM1 for $\sin x \to \pm \cos x$ and $\sin x \cos^2 x \to K \cos^3 x$

A1 $-\cos x - \left(-\frac{\cos^3 x}{3}\right)$ oe (not a multiple of this (unless recovered))

Alt 2

M1 Sets $u = \cos x$, finds $\frac{du}{dx} = \pm \sin x$ and makes a full substitution using both of these to get an integral in terms of u only. (Limits not needed for this mark).

dM1 for $au^2 - b \to Au^3 - bu$

A1 For reaching $\left[\frac{u^3}{3} - u\right]_1^{\frac{1}{2}}$ including correct limits or for undoing the substitution and reaching $\frac{\cos^3 x}{3} - \cos x$