9.

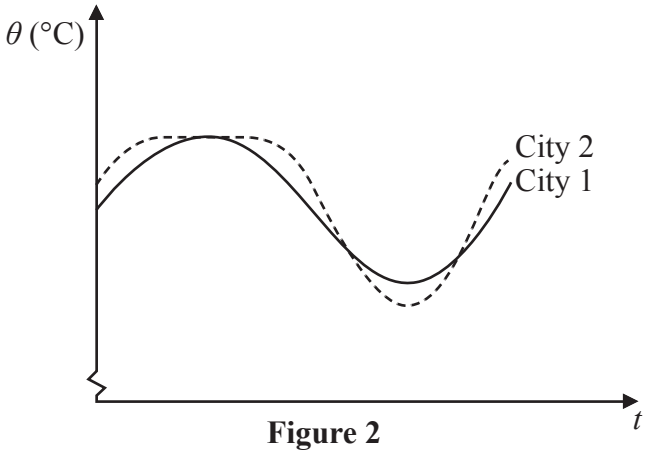

Figure 2

Figure 2 shows graphs modelling the temperature, in degrees Celsius, in two cities over the same 24-hour period from 8 am to 8 am.

The temperature, θ°C, in City 1 is modelled by the equation

$$\theta = 17 + 5\sin\left(\frac{\pi t}{12}\right)$$

where t is the number of hours after 8 am.

(a) Show that, according to the model, at 12 noon the temperature in City 1 is approximately 21.3°C.

(2)

The temperature, θ°C, in City 2 is modelled by the equation

$$\theta = 16 + 6\sin\left(\frac{\pi t}{12}\right) + 3\cos^2\left(\frac{\pi t}{12}\right)$$

where t is the number of hours after 8 am.

Use the equations of the models to answer parts (b) and (c).

When $t = T$ the temperature in each city is the same.

(b) Show that T satisfies the equation

$$3\sin^2\left(\frac{\pi T}{12}\right) - \sin\left(\frac{\pi T}{12}\right) - 2 = 0$$

(3)

(c) Hence find the values of T for which the temperature in each city is the same, giving the answers to 3 significant figures where necessary.
You must show all stages of your working.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(5)

(Total for Question 9 is 10 marks)

10. (i) Given that

- $f(n) = 168n - 119 - 24n^2$
- n is a positive integer less than 7

use proof by exhaustion to show that $f(n)$ is always a perfect square.

(3)

You may use the table on page 30 of the answer book to illustrate your answer.
The first 2 rows have been completed for you.

(ii) Given n is an integer with $n \geqslant r$, use the formula

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

to prove that

$$^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$$

You must show all stages of your working.

(4)

(Total for Question 10 is 7 marks)

TOTAL FOR PAPER IS 75 MARKS