8. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of $(k + 4x)^{12}$, where k is a constant. Give each term in simplest form.

(4)

Given that the coefficient of x^3 in this expansion is $\frac{55}{2}$

(b) find the value of k.

(2)

Using the value of k found in part (b) and the answer to part (a),

(c) find the coefficient of y^3 in the expansion of

$$\left(y + \frac{2}{y}\right)\left(k + 4y^2\right)^{12}$$

(3)

(Total for Question 8 is 9 marks)

9.

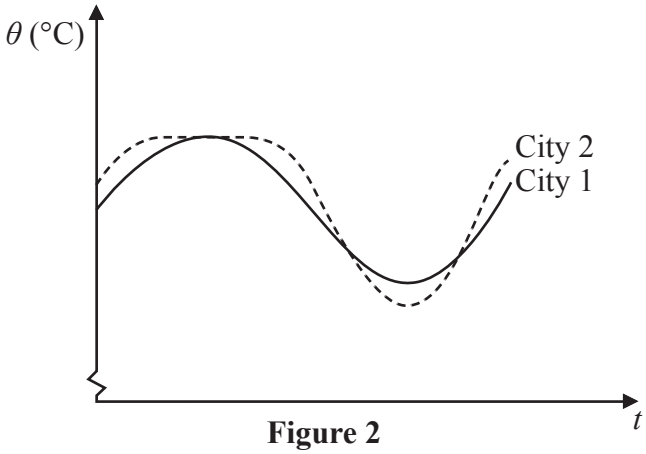

Figure 2

Figure 2 shows graphs modelling the temperature, in degrees Celsius, in two cities over the same 24-hour period from 8 am to 8 am.

The temperature, θ°C, in City 1 is modelled by the equation

$$\theta = 17 + 5\sin\left(\frac{\pi t}{12}\right)$$

where t is the number of hours after 8 am.

(a) Show that, according to the model, at 12 noon the temperature in City 1 is approximately 21.3°C.

(2)

The temperature, θ°C, in City 2 is modelled by the equation

$$\theta = 16 + 6\sin\left(\frac{\pi t}{12}\right) + 3\cos^2\left(\frac{\pi t}{12}\right)$$

where t is the number of hours after 8 am.

Use the equations of the models to answer parts (b) and (c).

When $t = T$ the temperature in each city is the same.

(b) Show that T satisfies the equation

$$3\sin^2\left(\frac{\pi T}{12}\right) - \sin\left(\frac{\pi T}{12}\right) - 2 = 0$$

(3)

(c) Hence find the values of T for which the temperature in each city is the same, giving the answers to 3 significant figures where necessary.
You must show all stages of your working.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(5)

(Total for Question 9 is 10 marks)