Question	Scheme	Marks
8(a)	$(x \pm 4)^2 ..(y \pm 6)^2 = ..$	**M1**
	Centre is $(4,-6)$	**A1**
	$r = \sqrt{16+36-27} = 5$	**A1**
		(3)
(b)	Deduces that the x coordinate of B is 8	**B1ft**
	$\frac{dy}{dx} = 0$ at $x = '8' \Rightarrow 2ax - \frac{3}{2} = 0 \Rightarrow 16a - \frac{3}{2} = 0$	**M1**
	$a = \frac{3}{32}$ *	**A1***
		(3)
(c)	$\int \frac{3}{32}x^2 - \frac{3}{2}x - 3\,dx = ..x^3 + ..x^2 + ..x$	**M1**
	$\int_0^{'8'} \frac{3}{32}x^2 - \frac{3}{2}x - 3\,dx = \left[\frac{1}{32}x^3 - \frac{3}{4}x^2 - 3x\right]_0^{'8'} = \left[\frac{8^3}{32} - \frac{3}{4}\times 8^2 - 3\times 8\right] - (0)$	**dM1**
	$= -56$	**A1**
	$X = (8,0)$, Area trapezium $OABX = \frac{1}{2}(3 + '9') \times '8' = ...$	**M1**
	Area semi-circle = $\frac{1}{2}\pi \times '5'^2 = \frac{'25'\pi}{2}$	**B1ft**
	Area R = $'56' - '48' + '\frac{25\pi}{2}'$	**dM1**
	$= 8 + \frac{25\pi}{2}$	**A1**
		(7)
		(13 marks)

(a)

M1: Attempt to complete the square witnessed by sight of $(x \pm 4)^2 ..(y \pm 6)^2 = ..$

Implied by a centre given as $(\pm 4, \pm 6)$

A1: Centre is $(4,-6)$ o.e. Condone brackets not being present

A1: Radius is 5, not $\sqrt{25}$. M1 must be awarded but may be scored following an incorrect centre $(\pm 4, \pm 6)$

(b)

B1ft: States or implies the correct x coordinate for point B.

Follow through on twice their x coordinate for their centre. It may be awarded from work in (c) or on the graph. It is implied by their top limit in their integral

M1: Differentiates, sets equal to 0 and substitutes their x at B to set up an equation in a.

Alternatives exist:

Alt I: Completes the square and sets x coordinate of vertex equal to their x coordinate of B and solves for a. Alternatively they find the line of symmetry and set equal to their 8

E.g. $y = a\left(\left(x - \frac{3}{4a}\right)^2 - ...\right) - 3 \Rightarrow \frac{3}{4a} = '8'$

Alt II: Works out the coordinates of B and substitutes into the parabola equation to find a

E.g. $A = (0, -3) \rightarrow \text{Centre} = (4, -6) \rightarrow B = (8, -9)$ so B is $(8, -9) \Rightarrow -9 = 64a - 12 - 3.$

Any point that lies on the parabola apart from $(0, -3)$ would be suitable. So, via symmetry

$A = (0, -3) \rightarrow B = (8, -9) \rightarrow (16, -3)$ lies on curve, $\Rightarrow -3 = 256a - 24 - 3$

A1*: Fully correct. Note that this is a printed answer and you must see an appropriate method

(c)

M1: Integrates the quadratic to the form $..x^3 + ..x^2 + ..x$. There is no requirement for a constant of integration or it to be simplified. This may be awarded from anywhere in their solution, even (a)

Condone slips on the coefficients of the given quadratic before integrating, allow even with 'ax^2'

dM1: Substitutes their 8 (2 × the x coordinate of their centre of C) and 0 into the integral and subtracts either way round. It is dependent upon having integrated to the form $..x^3 + ..x^2 + ..x$

The substitution of 0 may be implied which is fine. This can be implied by a correct value for their $\left[..x^3 + ..x^2 + ..x\right]_0^8$ but $\int_0^8 \frac{3}{32}x^2 - \frac{3}{2}x - 3\,dx = \pm 56$ is M0, dM0 as there is no algebraic integration

A1: Correct result of integrating and substituting, accept either 56 or -56 as they may realise areas must be positive. May be implied if not seen directly (allow if integration was correct and a correct attempt to substitute was made and used in working).

M1: Attempts the area of trapezium $OABX$ where X is their (8,0). It is dependent upon

- the '8' used must be 2 × the x coordinate of their centre of C
- the '9' used must be the y coordinate of the curve or circle at x = '8'

An alternative could be to integrate the equation of line AB between 0 and 8. Condone slips

B1ft: Correct semi-circle area following through their radius from (a). May be left unsimplified

dM1: Correct overall strategy for finding the required area.

Look for an attempt at the area of integral minus trapezium plus semi circle

It is dependent upon a correct method for the area of ALL shapes which must be combined correctly

A1: Correct answer.

Alt for first 4 marks: Candidate attempts to combine line AB and curve

M1: Attempts integral of (quadratic − line AB) either way around. Condone slips on equation of AB

Look for $...x^2 + ...x \rightarrow ...x^3 + ...x^2$ but condone $...x^2 + ...x + ... \rightarrow ...x^3 + ...x^2 + ...x$

dM1: Applies limits 0 and their 8 and subtracts either way round. See main method for conditions

A1: Correct answer = ±8 .

M1: For subtracting the equation of *AB* from the equation of the curve (or vice versa), integrating and applying limits. For example,

$$\pm\int_0^8 \left\{\left(\frac{3}{32}x^2 - \frac{3}{2}x - 3\right) - \left(-\frac{3}{4}x - 3\right)\right\}\mathrm{d}x = \pm\left[\frac{3}{96}x^3 - \frac{3}{8}x^2\right]_0^8$$

The method of finding the equation of line *AB* must be sound, but condone slips.

dM1: In this method it would be for adding together the area between the line and curve and the semi- circle

You may see alternatives where translations have been used, e.g. up 9 units where the trapezium becomes a triangle

Question	Scheme	Marks
9(a)	**States** that $x = 2r + 10$ o.e. **and** attempts $k\pi\, r^2 = k\pi\left(\frac{x-10}{2}\right)^2$ where $k = 1, \frac{1}{2}, \frac{1}{4}$	**M1**
	Area $= xy + 2\times\frac{\pi}{4}\times\left(\frac{x-10}{2}\right)^2 \Rightarrow A = xy + \frac{\pi}{8}(x-10)^2$ *	**A1***
		(2)
(b)	$A = 10000 \Rightarrow y = \frac{10000}{x} - \frac{\pi}{8x}(x-10)^2$ $\Rightarrow P = 2\left(\frac{10000}{x} - \frac{\pi}{8x}(x-10)^2\right) + \left(2+\frac{\pi}{2}\right)x - 5\pi$	**M1** **A1**
	$= \frac{20000}{x} - \frac{\pi}{4x}(x^2 - 20x + 100) + \left(2+\frac{\pi}{2}\right)x - 5\pi$ $= \frac{20000}{x} - \frac{\pi x}{4} + 5\pi - \frac{25\pi}{x} + \left(2+\frac{\pi}{2}\right)x - 5\pi = \frac{\ldots}{x} + (\ldots.)x + \ldots$	**M1**
	$\Rightarrow P = \frac{20000 - 25\pi}{x} + \left(2+\frac{\pi}{4}\right)x$ *	**A1***
		(4)
(c)	$\left(\frac{dP}{dx}\right) = -\frac{20000-25\pi}{x^2} + 2 + \frac{\pi}{4}$	**M1**
	$\frac{dP}{dx} = 0 \Rightarrow \frac{20000-25\pi}{x^2} = 2+\frac{\pi}{4} \Rightarrow x^2 = \frac{20000-25\pi}{2+\pi/4} \Rightarrow x = \ldots$	**dM1**
	$x = 84.6$	**A1**
		(3)
(d)	$\frac{d^2P}{dx^2} = \frac{40000-50\pi}{x^3} > 0$ for $x > 0$	**M1**
	As $\frac{d^2P}{dx^2} > 0$ so x gives a minimum value for the perimeter.	**A1**
		(2)
(11 marks)		

(a) This is a 'show that' and the answer is given, but condone *r = R*

M1: Look for a statement of $x = 2r + 10$ o.e, AND an attempt at the area of a full circle, semi-circle or quarter circle. Score for $\pi\left(\frac{x}{2}-5\right)^2$, $\frac{1}{2}\times\pi\left(\frac{x-10}{2}\right)^2$ or $\frac{1}{4}\times\pi\left(\frac{x-10}{2}\right)^2$ seen anywhere in (a)

A1*: Completes correctly to the given answer, no errors seen and suitable intermediate work. Look for

- A or area = being used at some point in the proof