**8.**

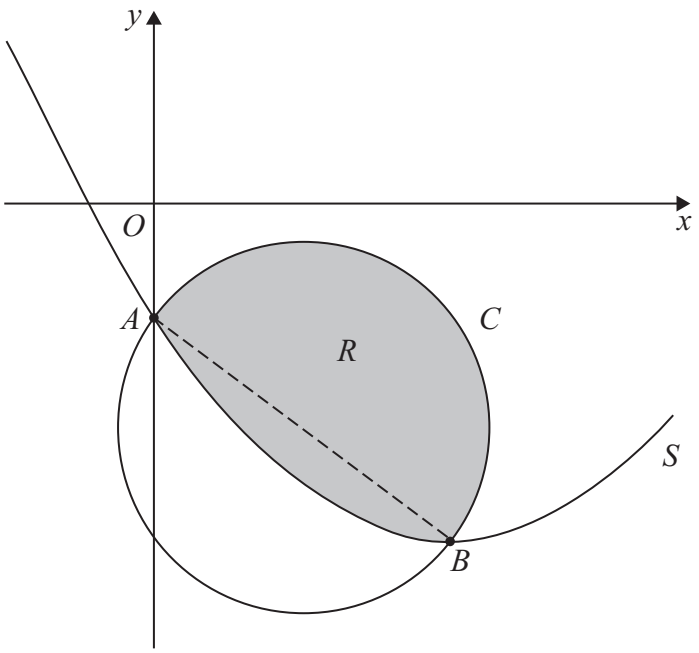


**Figure 3**

Figure 3 shows a sketch of a circle $C$ and a parabola $S$.

The circle $C$ has equation $x^2 + y^2 - 8x + 12y + 27 = 0$

(a) Find

(i) the coordinates of the centre of $C$

(ii) the radius of $C$

**(3)**

The parabola $S$

- has equation $y = ax^2 - \frac{3}{2}x - 3$
- intersects $C$ on the $y$-axis at the point $A(0, -3)$
- intersects $C$ again at the point $B$ where $AB$ is a diameter of $C$

Given that $B$ is the minimum point of $S$,

(b) show that $a = \frac{3}{32}$

**(3)**

The finite region $R$ is bounded by $S$ and $C$ and is shown shaded in Figure 3.

(c) Use algebraic integration to find the exact area of $R$.

**(7)**

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**Question 8 continued**

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**(Total for Question 8 is 13 marks)**

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**9.**

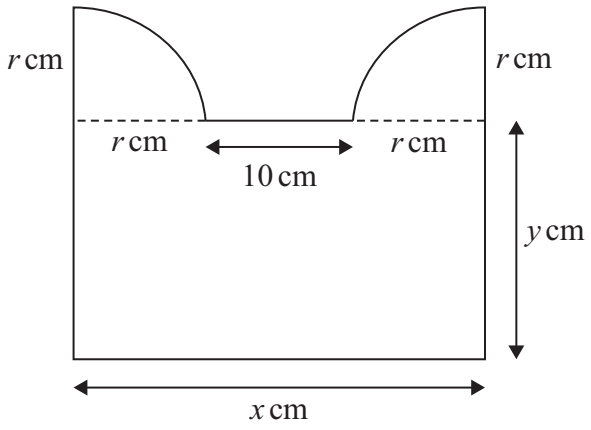


**Figure 4**

Figure 4 shows a sketch of a design for a logo.

The design consists of a rectangle of base $x$ cm and height $y$ cm with two quarter circles of equal radius $r$ cm joined to the top edge with a gap of 10 cm between the nearest point of each quarter circle, as shown in Figure 4.

(a) Show that the area, $A$ cm$^2$, of the design is given by

$$A = xy + \frac{\pi}{8}(x - 10)^2$$

**(2)**

Given that

- the total area of the design is one square metre
- the perimeter, $P$ cm, of the design is given by $P = 2y + \left(2 + \frac{\pi}{2}\right)x - 5\pi$

(b) show that

$$P = \frac{20000 - 25\pi}{x} + \left(2 + \frac{\pi}{4}\right)x$$

**(4)**

(c) Use algebraic differentiation to find the value of $x$ that gives the minimum value for $P$, giving the answer to 3 significant figures.

**(3)**

By considering $\frac{\mathrm{d}^2P}{\mathrm{d}x^2}$

(d) justify that the value for $x$ found in part (c) gives the minimum value for $P$.

**(2)**

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