Question Number	Scheme	Marks
8(a)	$\dfrac{\cos\theta}{2\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$	M1
	$\cos^2\theta = 2\sin\theta(1+\sin\theta) \Rightarrow 1-\sin^2\theta = 2\sin\theta(1+\sin\theta)$	dM1
	$\Rightarrow 1-\sin^2\theta = 2\sin\theta+2\sin^2\theta \Rightarrow 3\sin^2\theta+2\sin\theta-1=0$ *	A1*
		(3)
(b)	$3\sin^2\theta+2\sin\theta-1=0 \Rightarrow \sin\theta=\dfrac{1}{3},\ \{-1\}$	B1
	Uses $\cos^2\theta \equiv 1-\sin^2\theta \Rightarrow \cos^2\theta = 1-\left("\dfrac{1}{3}"\right)^2$	M1
	$\Rightarrow \{\cos\theta =\}\ \dfrac{2\sqrt{2}}{3}$	A1
		(3)
(c)	Attempts common ratio $= \dfrac{\cos\theta}{2\sin\theta} = \dfrac{\left("\dfrac{2\sqrt{2}}{3}"\right)}{\left("\dfrac{2}{3}"\right)} = \ldots \{\sqrt{2}\}$	M1
	Finds $ar^{10} = "\dfrac{2}{3}" \times \left("\sqrt{2}"\right)^{10} = \ldots$	dM1
	$\dfrac{64}{3}$	A1
		(3)
		(9 marks)

(a)

M1: Uses common ratios and forms a correct equation in terms of θ, e.g.,

$u_1u_3 = u_2{}^2 \Rightarrow \cos^2\theta = 2\sin\theta(1+\sin\theta)$ or e.g. $1+\sin\theta = 2\sin\theta\left(\dfrac{\cos\theta}{2\sin\theta}\right)^2$

dM1: Cross multiplies (if necessary) and uses $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ to form a quadratic equation in $\sin\theta$
Dependent on the previous method mark.

A1*: Expands the brackets and proceeds correctly to $3\sin^2\theta+2\sin\theta-1=0$ with no errors.
They must have a correct intermediate line i.e. expanded any brackets before the given answer.
Condone the occasional slip in missing or incorrect arguments or use of e.g. $\sin\theta^2$ but penalise consistent errors in the main body of their work (not their side working). Requires the = 0.

(b)

B1: Solves the given equation find $\sin\theta = \dfrac{1}{3}$ Allow the use of a calculator to solve the 3TQ here.
May be scored in part (a). Ignore any reference to $\sin\theta = -1$ for this mark only.
Not scored for e.g. $x = \sin\theta$ followed by $x = \dfrac{1}{3}$ unless used later in e.g. $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$

M1: Uses $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ to achieve an exact value for $\cos^2\theta$, i.e., $\cos^2\theta = 1-\sin^2\theta = 1-\left("\frac{1}{3}"\right)^2$ or an exact value for $\cos\theta$ which may come directly from a right-angled triangle with sides that are multiples of 1, $2\sqrt{2}$ (or $\sqrt{8}$) and 3 (or otherwise for their $\sin\theta = ...$ provided $\sin\theta \neq \pm 1$ or 0)

Condone if they return to $\frac{\cos\theta}{2\sin\theta} = \frac{1+\sin\theta}{\cos\theta} \Rightarrow \frac{\cos\theta}{2\left(\frac{1}{3}\right)} = \frac{1+\left(\frac{1}{3}\right)}{\cos\theta} \Rightarrow \cos^2\theta = ...$ for this mark.

Alternatively, condone if they find *r* through e.g. $u_3 = u_1 r^2 \Rightarrow r^2 = \frac{1+"\frac{1}{3}"}{2\left("\frac{1}{3}"\right)} = ...\{2\}$ **and** go on to find $u_2 = u_1 r = 2\left("\frac{1}{3}"\right)"\sqrt{2}" = ...$ Note that this approach will also score the first M1 in part (c).

Attempting $\cos\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$ $\{= 0.9428...\}$ is not an acceptable method.

A1: $\{\cos\theta =\}$ $\frac{2\sqrt{2}}{3}$ only (o.e e.g. $\{\cos\theta =\}$ $\frac{\sqrt{8}}{3}$) and ISW after a correct answer.

(c) **No marks can be scored in part (c) from use of $\sin\theta = \pm 1$ as these trivialise the solutions. Similarly, no marks can be scored if their $\sin\theta$ or $\cos\theta$ are inexact.**

M1: Attempts common ratio using either $\frac{\cos\theta}{2\sin\theta} = \frac{"\frac{2\sqrt{2}}{3}"}{2\left("\frac{1}{3}"\right)} = ...\left\{\sqrt{2}\right\}$ or $\frac{1+\sin\theta}{\cos\theta} = \frac{1+"\frac{1}{3}"}{"\frac{2\sqrt{2}}{3}"} = ...\left\{\frac{2}{\sqrt{2}}\right\}$ as far as a value but may be implied. Note this may be seen in (b).

May come from equivalent trigonometric expressions e.g. $\frac{1}{2}\cot\theta = \frac{2\sqrt{2}}{2}$ $\left\{= \sqrt{2}\right\}$

Alternatively, attempts r^2 through e.g. $u_3 = u_1 r^2 \Rightarrow r^2 = \frac{1+"\frac{1}{3}"}{2\left("\frac{1}{3}"\right)} = ...\{2\}$ which may be seen in (b).

dM1: Finds $ar^{10} = "\frac{2}{3}"\times\left("\sqrt{2}"\right)^{10} = ...$ where *a* is their $2\sin\theta$

Dependent on the previous method mark but note that the marks may be scored simultaneously.

You may see $ar^{10} = "\frac{2}{3}"\times("2")^5 = ...$ which is also acceptable, as are other correct alternatives.

A1: $\frac{64}{3}$ o.e. e.g. $21\frac{1}{3}$ Do not allow e.g. 21.33 or 21.33… but allow $21.\dot{3}$

Must follow a correct answer in part (b) unless they only use the **alternative** method which does not require a value for cos*θ*.

but do **not** allow this mark if they clearly use incorrect e.g. $\cos\theta = -\frac{2\sqrt{2}}{3}$ or $\cos\theta =$ awrt 0.943

Note you may see $ar^{10} = 2\sin\theta\times\left(\frac{\cos\theta}{2\sin\theta}\right)^{10} = \frac{64}{3}$ which will score full marks provided it follows a correct exact answer in part (b).

Do not ISW if they e.g. go on to find the sum of the first 11 terms.

Question Number	Scheme	Marks
9.	**Way 1** $\log_4 \frac{a}{b} = 3$ **or** $\log_4 a = \log_4 64b$ $\frac{a}{b} = 64$ Solves $ab = 25$ with $\frac{a}{b} = k$ and proceeds to $a =\ldots$ **or** $b =\ldots$ e.g. $a^2 = 1600 \Rightarrow a = ...$ $a = 40$ **and** $b = \frac{5}{8}$	M1 A1 dM1 A1 **(4)** **(4 marks)**

Note: Condone the absence of base 4 throughout.

If candidates write $\log_4 a - \log_4 b = 3 \rightarrow \frac{\log_4 a}{\log_4 b} = 3 \rightarrow \frac{a}{b} = 4^3$ **allow maximum 1010.**

M1: **Correct** use of addition or subtraction law for logs e.g. $\log_4 a - \log_4 b \rightarrow \log_4 \frac{a}{b}$

or $\log_4 a = 3 + \log_4 b = \log_4 p + \log_4 b = \log_4 pb$ (for some constant $p > 0$ with $p \neq 3$)

or (**Way 2**) $\log_4 ab \rightarrow \log_4 a + \log_4 b$

Alternatively, via **Way 3**, achieves a correct $a = 4^{3+\log_4 b}$ or $b = 4^{\log_4 a - 3}$

In **Way 4**, attempts to make a or b the subject of the first equation and substitutes into the second equation.

Not scored for just e.g. $3 = \log_4 64$ or for e.g. $\log_4 a - \log_4 b = 3$ going directly to $\frac{a}{b} = 3$

A1: **Way 1**: Achieves a second correct equation in terms of a and b with logs removed i.e. $\frac{a}{b} = 4^3$ o.e.

Way 2: Achieves a correct equation only in a **or** only in b which may still have logs.
e.g. $2\log_4 a = 3 + \log_4 25$ **or** $2\log_4 b = \log_4 25 - 3$

Way 3: Achieves a correct equation only in a **or** only in b which may still have logs.
e.g. $b \times 4^{3+\log_4 b} = 25$ or $a \times 4^{\log_4 a - 3} = 25$

Way 4: Writes e.g. $a = \frac{25}{b}$ and substitutes into the second equation to achieve a correct equation only in a or only in b e.g. $\log_4 \frac{25}{b} - \log_4 b = 3$ (proceed to mark as in **Way 2**).

Note that in all approaches they may have already replaced 3 with $\log_4 64$