Question Number	Scheme	Marks
7 (a)	$100 = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$	M1
	$y = \frac{100}{x} - \frac{\pi x}{8}$	A1
	$P = 2y + x + \frac{1}{2}\pi x = \frac{200}{x} - \frac{\pi x}{4} + x + \frac{1}{2}\pi x$	dM1
	$P = \frac{1}{4}x(4+\pi) + \frac{200}{x}$ *	A1*
		(4)
(b)	$\left\{\frac{\mathrm{d}P}{\mathrm{d}x} =\right\} 1 + \frac{\pi}{4} - \frac{200}{x^2}$	B1
	$1 + \frac{\pi}{4} - \frac{200}{x^2} = 0 \Rightarrow x^2 = \frac{800}{4+\pi}$	M1
	$\Rightarrow x = \sqrt{\frac{800}{4+\pi}}$	A1
		(3)
(c)	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{\lambda}{x^3} = \frac{\lambda}{"10.6"^3} = \ldots$ or makes reference to the sign of $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{\lambda}{x^3}$	M1
	e.g. $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{400}{x^3}$ so (since $x > 0$) $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} > 0 \Rightarrow$ minimum	A1
		(2)
(d)	Substitutes $x = \sqrt{\frac{800}{4+\pi}} \approx 10.6$ into $P = \frac{200}{x} + \frac{1}{4}x(4+\pi) = \ldots$	M1
	= awrt 37.8 {m}	A1
		(2)
		(11 marks)

(a)

M1: Sets $100 = xy + kx^2$ where k is a non-zero constant e.g. $100 = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ (may be implied by later work)

A1: Achieves $y = \frac{100}{x} - \frac{\pi x}{8}$ or equivalent correct y in terms of x e.g. $y = 100x^{-1} - \frac{\pi x}{8}$ or $y = \frac{800 - \pi x^2}{8x}$

dM1: Substitutes their y in terms of x into their expression for the perimeter. Don't be concerned about the mechanics of rearranging the subject for this mark.
Dependent on the previous method mark.

A1*: Full proof showing all relevant lines with no errors or omissions.
There is no need to collect terms in x prior to factorisation but there must be evidence of correct substitution of a correct y in terms of x into a correct P in terms of y and x before the given answer.
The $P =$ (allow perimeter = or even perimeter:) must be present at some stage but accept if it is first seen in the answer line.
Condone $200x^{-1}$ in place of $\frac{200}{x}$ and allow the terms in any order.

(b) Note: Full marks in (b) can be scored from use of $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\mathrm{f}'(x)$ in place of $\frac{\mathrm{d}P}{\mathrm{d}x}$ (or P')

B1: Fully correct derivative $\left\{\frac{\mathrm{d}P}{\mathrm{d}x}=\right\}1+\frac{\pi}{4}-\frac{200}{x^2}$ o.e e.g. $\left\{\frac{\mathrm{d}P}{\mathrm{d}x}=\right\}\frac{4+\pi}{4}-200x^{-2}$

The indices must be processed, i.e., not $\left\{\frac{\mathrm{d}P}{\mathrm{d}x}=\right\}1+\frac{\pi}{4}-200x^{-1-1}$

M1: Sets their $\frac{\mathrm{d}P}{\mathrm{d}x}=0$ which must be of the form $A-Bx^{-2},\ A,B>0$ **and** proceeds at least to $x^{\pm 2}=C,\ C>0$ which may be implied by their solution of e.g. $D=Ex^{-2},\ D\times E>0$

Alternatively, they may use the quadratic formula. Condone the use of an inequality for this mark.

A1: cao $x=\sqrt{\frac{800}{4+\pi}}$ o.e. such as $\frac{20\sqrt{2}}{\sqrt{4+\pi}}$ or $\frac{20\sqrt{8+2\pi}}{4+\pi}$ and ISW after an acceptable answer.

It must be exact and in a simple form and **not** a decimal 10.58… or embedded fractions e.g. $\sqrt{\frac{200}{1+\frac{\pi}{4}}}$ or $\frac{10\sqrt{2}}{\sqrt{1+\frac{\pi}{4}}}$ are A0 $\quad\pm\sqrt{\frac{800}{4+\pi}}$ is also A0. Must now be an equality and not e.g. $x>\sqrt{\frac{800}{4+\pi}}$

(c) **Notes: All marks in (c) and (d) can be scored if *x* = 10.6 or better.**
Mark part (c) and part (d) together.

M1: For an attempt to

- Either substitute their answer to part (b) into $\frac{\mathrm{d}^2P}{\mathrm{d}x^2}=\frac{\lambda}{x^3}=\ldots$ and find a value (you may need to check if this is implied)
- Or make reference to the sign of $\frac{\mathrm{d}^2P}{\mathrm{d}x^2}=\frac{\lambda}{x^3}$
- Alternatively, they may substitute *x* values either side of their value found in part (b) into their $\frac{\mathrm{d}P}{\mathrm{d}x}$ to find two values for $\frac{\mathrm{d}P}{\mathrm{d}x}$

A1: cso Requires a correct $\frac{\mathrm{d}^2P}{\mathrm{d}x^2}=\frac{400}{x^3}$, a value of 10.6 or better in part (b), a statement and a reason, e.g. $\frac{\mathrm{d}^2P}{\mathrm{d}x^2}=\frac{400}{x^3}>0\Rightarrow$ minimum

If they calculate the value of the 2nd derivative it must be 0.3 (1sf) or better and they must state > 0

If using the 2nd derivative they must use the notation $\frac{\mathrm{d}^2P}{\mathrm{d}x^2}$ or P'' for this mark (condone $\frac{\mathrm{d}P^2}{\mathrm{d}^2x}$)

In the alternative, the two values substituted in must be either side of $\sqrt{\frac{800}{4+\pi}}$, the values of $\frac{\mathrm{d}P}{\mathrm{d}x}$ must be correct to 1sf, they must use the notation $\frac{\mathrm{d}P}{\mathrm{d}x}$ (or P') and they must conclude that the gradient changes from negative to positive implying a minimum point (or equivalent).

(d)

M1: Substitutes their $x = \sqrt{\dfrac{800}{4+\pi}} \approx 10.6$ into the **given** $P = \dfrac{1}{4}x\left(4+\pi\right)+\dfrac{200}{x} = ...$ or otherwise correct P

You may need to check their answer if they do not show the substitution.

A1: awrt 37.8{m} and must be a decimal.

Note that if **only** awrt 37.8 is given (i.e. no substitution shown) then they must have used calculus to find awrt 10.6 or better in part (b) to score this mark, otherwise score M0A0.

Question Number	Scheme	Marks
8(a)	$\dfrac{\cos\theta}{2\sin\theta} = \dfrac{1+\sin\theta}{\cos\theta}$	M1
	$\cos^2\theta = 2\sin\theta(1+\sin\theta) \Rightarrow 1-\sin^2\theta = 2\sin\theta(1+\sin\theta)$	dM1
	$\Rightarrow 1-\sin^2\theta = 2\sin\theta+2\sin^2\theta \Rightarrow 3\sin^2\theta+2\sin\theta-1=0$ *	A1*
		(3)
(b)	$3\sin^2\theta+2\sin\theta-1=0 \Rightarrow \sin\theta=\dfrac{1}{3},\ \{-1\}$	B1
	Uses $\cos^2\theta \equiv 1-\sin^2\theta \Rightarrow \cos^2\theta = 1-\left("\dfrac{1}{3}"\right)^2$	M1
	$\Rightarrow \{\cos\theta =\}\ \dfrac{2\sqrt{2}}{3}$	A1
		(3)
(c)	Attempts common ratio $= \dfrac{\cos\theta}{2\sin\theta} = \dfrac{\left("\dfrac{2\sqrt{2}}{3}"\right)}{\left("\dfrac{2}{3}"\right)} = \ldots \{\sqrt{2}\}$	M1
	Finds $ar^{10} = "\dfrac{2}{3}" \times \left("\sqrt{2}"\right)^{10} = \ldots$	dM1
	$\dfrac{64}{3}$	A1
		(3)
		(9 marks)

(a)

M1: Uses common ratios and forms a correct equation in terms of θ, e.g.,

$u_1u_3 = u_2{}^2 \Rightarrow \cos^2\theta = 2\sin\theta(1+\sin\theta)$ or e.g. $1+\sin\theta = 2\sin\theta\left(\dfrac{\cos\theta}{2\sin\theta}\right)^2$

dM1: Cross multiplies (if necessary) and uses $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ to form a quadratic equation in $\sin\theta$
Dependent on the previous method mark.

A1*: Expands the brackets and proceeds correctly to $3\sin^2\theta+2\sin\theta-1=0$ with no errors.
They must have a correct intermediate line i.e. expanded any brackets before the given answer.
Condone the occasional slip in missing or incorrect arguments or use of e.g. $\sin\theta^2$ but penalise consistent errors in the main body of their work (not their side working). Requires the = 0.

(b)

B1: Solves the given equation find $\sin\theta = \dfrac{1}{3}$ Allow the use of a calculator to solve the 3TQ here.
May be scored in part (a). Ignore any reference to $\sin\theta = -1$ for this mark only.
Not scored for e.g. $x = \sin\theta$ followed by $x = \dfrac{1}{3}$ unless used later in e.g. $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$