7.

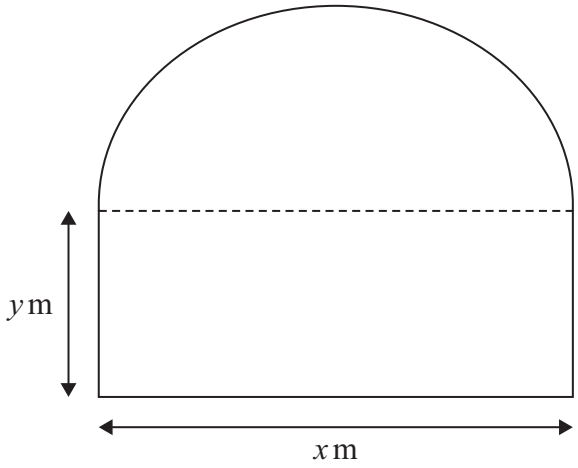


Diagram **NOT** drawn to scale

Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows the plan view of a garden. The shape of this garden consists of a rectangle joined to a semicircle.

The rectangle has length x metres and width y metres.

The area of the garden is $100\,\text{m}^2$.

(a) Show that the perimeter, P metres, of the garden is given by

$$P = \frac{1}{4}x(4 + \pi) + \frac{200}{x}$$

(4)

(b) Use calculus to find the exact value of x for which the perimeter of the garden is a minimum.

(3)

(c) Justify, by further use of calculus, that the value of x found in part (b) gives a minimum value for P.

(2)

(d) Find the minimum perimeter of the garden, giving your answer in metres to one decimal place.

(2)

(Total for Question 7 is 11 marks)

8. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given the first three terms of a geometric sequence are

$$2\sin\theta,\ \cos\theta \text{ and } 1+\sin\theta$$

where θ is a constant

(a) show that

$$3\sin^2\theta + 2\sin\theta - 1 = 0$$

(3)

Given that θ is acute,

(b) use trigonometric identities to find the exact value of the second term in the sequence.

(3)

(c) find the value of the 11th term in the sequence.

(3)

(Total for Question 8 is 9 marks)

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9. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given that a and b are positive constants, solve the simultaneous equations

$$ab = 25$$

$$\log_4 a - \log_4 b = 3$$

Show each step of your working, giving exact values for a and b.

(4)

(Total for Question 9 is 4 marks)
