Question Number	Scheme	Marks
6.(a)	{Mid-point of XY =} (3, 7)	B1
	gradient $XY = \frac{11-3}{6-0}$ $\left\{=\frac{4}{3}\right\}$	M1
	$y - "7" = "-\frac{3}{4}"(x - "3")$	dM1
	$3x + 4y - 37 = 0$	A1
		(4)
(b)	Substitute $y = 10$ into their line equation to give $x = \ldots$	M1
	$x = -1$	A1
		(2)
(c)	e.g. $r^2 = ("-1" - 0)^2 + (10-3)^2$ or $r = \sqrt{("-1"-6)^2 + (10-11)^2}$	M1
	$(x+1)^2 + (y-10)^2 = 50$	dM1, A1
		(3)
(d)	Full method to find W or e.g. $(..., 9)$ **or** $("-8", ...)$	M1 (B1 on EPEN)
	$(-8, 9)$	A1 (B1 on EPEN)
		(2)
		(11 marks)

Note: Throughout the question be generous with parts that are mislabelled or not labelled at all, provided it is clear what their intention is.

(a)

B1: Correct coordinates for {M =} (3, 7)

M1: Correct method for gradient of $XY = \frac{11-3}{6-0}$ $\left\{=\frac{4}{3}\right\}$

An alternative is to set up and solve an equation e.g. $11 = 6m + 3$ leading to $m = \ldots$

dM1: Correct method for the equation of the line through Z and their M (which must not be X or Y).

Look for use of the negative reciprocal of their $\frac{4}{3}$ with their (3, 7), e.g., $y - "7" = "-\frac{3}{4}"(x - "3")$

If using $y = mx + c$ they must proceed as far as $c = \ldots$ Dependent on the previous method mark.

A1: $3x + 4y - 37 = 0$ but allow any integer multiple of this and any order of terms.

The = 0 must be present. $3x + 4y = 37$ is A0 without also giving an alternative in the required form.

(b)

M1: Substitutes $y = 10$ into their $3x + 4y - 37 = 0$ (or earlier form of this equation) and solves to find a value for x

or Alt uses the fact that $XZ = YZ$ to give $x^2 + 7^2 = (x-6)^2 + 1^2$ and solves by expanding and then cancelling the x^2 terms to find a value for x

A1: cao $x = -1$ Note: if they use an equation from part (a) it must be correct.

If using **Alt** then you do not need to check their equation in part (a).

(c)

M1: Valid method to find the radius or radius^2, usually using Pythagoras with $("-1", 10)$ and either X or Y. Alternatively, uses $MZ^2 + MY^2 = r^2$ or $MZ^2 + MX^2 = r^2$

dM1: Correct method for the equation of the circle $(x\pm"-1")^2+(y\pm10)^2="\sqrt{50}"^2$ using their radius found using a valid method and condoning sign errors. Dependent on the first method mark.

A1: cao $(x+1)^2+(y-10)^2=50$ must be simplified. Do not allow e.g. $(x+1)^2+(y-10)^2=\sqrt{50}^2$

This mark is dependent on scoring the A1 in part (b) because we require that $x = -1$ comes from correct working.

ISW after a correct answer is seen e.g. if they go on to attempt to expand the brackets.

Those who fortuitously achieve a correct equation using $x = 1$ will score A0.

(d)

M1: For a full **method** as far as a value for x **or** y which *would* lead to a correct value for either coordinate of W following from their answers to previous parts of the question.

May be implied by one correct coordinate following through on their $x=-1$ using the main scheme, so either $(...,9)$ or $("-8",...)$. Follow through on $6-2\times(6-("-1"))$ for the x coordinate.

Allow this mark if they slip and write (9, –8).

Note that there are many ways to find the coordinates of W.

In any of these approaches, condone slips in copying or arithmetic mistakes but the core method must be valid, so any quadratic equation created **must** be solved using an appropriate strategy which may be by calculator. Do not be concerned by how they label any lines e.g. mislabel XW as YW.

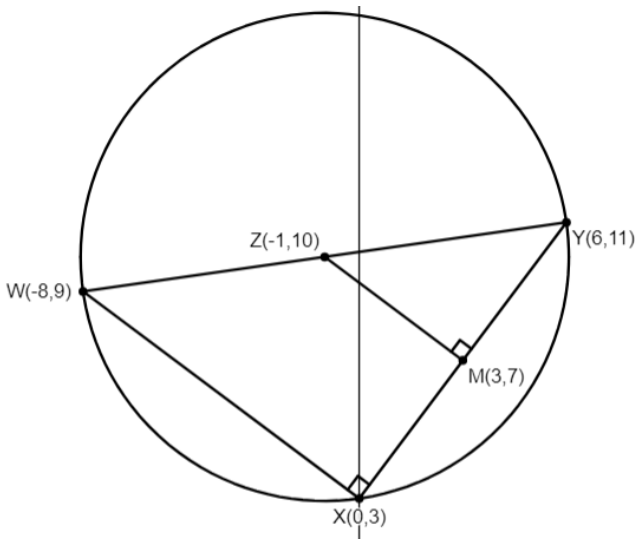
Some examples are:

- Using $\overrightarrow{OY}+2\overrightarrow{YZ}$ (as in the main scheme).
- Finding the equation of XW $\left(y-3="-\frac{3}{4}"x\right)$ and solving simultaneously with the equation of their circle.
- Finding the equation of YW $\left(y-11="\frac{1}{7}"(x-6)\text{ or }y-10="\frac{1}{7}"(x-"-1")\right)$ and solving simultaneously with the equation of their circle.
- Finding the equation of XW and the equation of YW (as above) and solving simultaneously.
- Finding $\overrightarrow{OX}+2\overrightarrow{MZ}$
- Using Pythagoras $XY^2 + XW^2 = (2r)^2$ with $W(a, b)$ lying on any of XW, YW or the circle.

Any of these approaches may let $W(a, b)$ first: e.g. in bullet 2, let $W(a, b)$ then gradient of WX is $\frac{b-3}{a}=-"\frac{3}{4}"$ leading to $a=4-\frac{4}{3}b$ which can then be substituted into the equation of the circle or their YW.

A1: cao $(-8, 9)$ and no others seen, i.e., if two possible pairs of coordinates are given then A0.

This mark is dependent on scoring the A1 in part (b) because we require that $x = -1$ comes from correct working.



Question Number	Scheme	Marks
7 (a)	$100 = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$	M1
	$y = \frac{100}{x} - \frac{\pi x}{8}$	A1
	$P = 2y + x + \frac{1}{2}\pi x = \frac{200}{x} - \frac{\pi x}{4} + x + \frac{1}{2}\pi x$	dM1
	$P = \frac{1}{4}x(4+\pi) + \frac{200}{x}$ *	A1*
		(4)
(b)	$\left\{\frac{\mathrm{d}P}{\mathrm{d}x} =\right\} 1 + \frac{\pi}{4} - \frac{200}{x^2}$	B1
	$1 + \frac{\pi}{4} - \frac{200}{x^2} = 0 \Rightarrow x^2 = \frac{800}{4+\pi}$	M1
	$\Rightarrow x = \sqrt{\frac{800}{4+\pi}}$	A1
		(3)
(c)	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{\lambda}{x^3} = \frac{\lambda}{"10.6"^3} = \ldots$ or makes reference to the sign of $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{\lambda}{x^3}$	M1
	e.g. $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{400}{x^3}$ so (since $x > 0$) $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} > 0 \Rightarrow$ minimum	A1
		(2)
(d)	Substitutes $x = \sqrt{\frac{800}{4+\pi}} \approx 10.6$ into $P = \frac{200}{x} + \frac{1}{4}x(4+\pi) = \ldots$	M1
	= awrt 37.8 {m}	A1
		(2)
		(11 marks)

(a)

M1: Sets $100 = xy + kx^2$ where k is a non-zero constant e.g. $100 = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$ (may be implied by later work)

A1: Achieves $y = \frac{100}{x} - \frac{\pi x}{8}$ or equivalent correct y in terms of x e.g. $y = 100x^{-1} - \frac{\pi x}{8}$ or $y = \frac{800 - \pi x^2}{8x}$

dM1: Substitutes their y in terms of x into their expression for the perimeter. Don't be concerned about the mechanics of rearranging the subject for this mark.
Dependent on the previous method mark.

A1*: Full proof showing all relevant lines with no errors or omissions.
There is no need to collect terms in x prior to factorisation but there must be evidence of correct substitution of a correct y in terms of x into a correct P in terms of y and x before the given answer.
The $P =$ (allow perimeter = or even perimeter:) must be present at some stage but accept if it is first seen in the answer line.
Condone $200x^{-1}$ in place of $\frac{200}{x}$ and allow the terms in any order.