Question Number	Scheme	Marks
7.(a)	$y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44$ $y = x^3 - 4x^{\frac{5}{2}} - kx^{\frac{1}{2}} + 28x - 44 \Rightarrow \left(\frac{dy}{dx}\right) = 3x^2 - 10x^{\frac{3}{2}} - \frac{k}{2}x^{-\frac{1}{2}} + 28$	M1, A1 **(2)**
(b)	Subs $x = 9$ into $\frac{dy}{dx}$ and sets = 0 $\Rightarrow 3\times 81 - 10\times 27 - \frac{k}{6} + 28 = 0$ $243 - 270 - \frac{k}{6} + 28 = 0 \Rightarrow \frac{k}{6} = 1 \Rightarrow k = 6$ *	M1 A1* **(2)**
(c)	$\int x^3 - 4x^{\frac{5}{2}} - 6x^{\frac{1}{2}} + 28x - 44\,dx = \frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x$ Correct value of $y = -53$ at T Shaded area = $"53"\times 9 + \left[\frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x\right]_0^9$ = 248	M1, A1, A1 B1 dM1 A1 **(6)** **(10 marks)**

(a)

M1: Attempts to differentiate. Look for **two** correct terms, one of which could be $-44 \to 0$.
Allow the coefficients to be un-simplified but the indices must be processed

A1: $\left(\frac{dy}{dx}\right) = 3x^2 - 10x^{\frac{3}{2}} - \frac{k}{2}x^{-\frac{1}{2}} + 28$ or other simplified equivalent. ISW after a correct answer

(b)

M1: Substitutes $x = 9$ into their $\frac{dy}{dx}$ and sets = 0. Look for embedded values, or 4 simplified terms set = 0

For the M1 only, allow a substitution of $x = 9$ into their $\frac{dy}{dx}$ which develops into an equation that implies $\frac{dy}{dx}$ = 0. E.g. $\frac{dy}{dx} = 3\times 9^2 - 10\times 9^{\frac{3}{2}} - \frac{k}{2}\times 9^{-\frac{1}{2}} + 28 = \frac{k}{6} - 1$ So $\frac{k}{6} = 1$

Alternatively substitutes $x = 9$ and $k = 6$ into their $\frac{dy}{dx}$ and finds the value of $\frac{dy}{dx}$ with conditions as above.

A1*: Shows that $k = 6$

Via the main method you must see

- an un-simplified equation such as $3\times 81 - 270 - \frac{1}{6}k + 28 = 0$
- a simplified equation such as $\frac{k}{6} = 1$ o.e.
- and the given answer $k = 6$

Via the alternative method you must see

- an un-simplified expression for $\frac{\mathrm{d}y}{\mathrm{d}x} = 3\times 81 - 270 - \frac{1}{6}\times 6 + 28$
- before proceeding to $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$
- followed by a conclusion stating that there is a stationary point when $k = 6$

(c) Main method: Finds both areas and subtracts

M1: Attempts to integrate. Look for two correct indices which must be processed

A1: Two correct terms but allow the coefficients to be left un-simplified

A1: $\frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x$ allowing coefficients to be left un-simplified.

Condone an additional +c

B1: $y = -53$ at *T* May be awarded at any point in the question, even from the diagram

dM1: Look for a FULL method that involves (area of rectangle –area under curve) either way around to find a value for the area of *R*. Condone slips but it is dependent upon the previous M.

The area of the rectangle must be found using $9\times$ (an attempt at the *y* value of the curve at *x* = 9), and not for instance using 9×44 as 44 is the intersection of the curve with the *y* -axis.

You don't need to see working for the evaluation of $\left[\frac{1}{4}x^4 - \frac{8}{7}x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + 14x^2 - 44x\right]_0^9$

A1: AWRT 248. Allow the exact answer $\frac{6939}{28}$ Note that -248 is A0

(c) Alt method via integration of line – curve (or curve –line)

This could also be attempted by translating the curve upwards by 53 units

The following example shows a correct line – curve approach

$$\int '-53' - \left(x^3 - 4x^{\frac{5}{2}} - 6x^{\frac{1}{2}} + 28x - 44\right)\mathrm{d}x = \int -9 - x^3 + 4x^{\frac{5}{2}} + 6x^{\frac{1}{2}} - 28x\,\mathrm{d}x$$

$$-9x - \frac{1}{4}x^4 + \frac{8}{7}x^{\frac{7}{2}} + 4x^{\frac{3}{2}} - 14x^2$$

Shaded area = $-9\times 9 - \frac{1}{4}\times 9^4 + \frac{8}{7}\times 9^{\frac{7}{2}} + 4\times 9^{\frac{3}{2}} - 14\times 9^2 = \frac{6939}{28}$

Area to 3sf = 248 which must be +ve

M1: Attempts to integrate an expression of the form $\pm\left(-x^3+4x^{\frac{5}{2}}+6x^{\frac{1}{2}}-28x\pm p\right)$.

Look for two correct indices which must be processed. Condone bracketing errors

Allow even with p = 97 or 0

A1: Two correct terms of $\pm\left(-\frac{1}{4}x^4+\frac{8}{7}x^{\frac{7}{2}}+4x^{\frac{3}{2}}-14x^2\pm px\right)$ but allow the coefficients to be left un-simplified. Follow through on their value of p which may be 97 or 0

A1: Correct integration. So look for $\pm\left(-\frac{1}{4}x^4+\frac{8}{7}x^{\frac{7}{2}}+4x^{\frac{3}{2}}-14x^2\pm px\right)$, allowing the coefficients to be left un-simplified. Follow through on their value of p which may be 97 or 0. Condone an additional + c

B1: For $y=-53$ at T. May be awarded at any point in the question, even from the diagram

dM1: Attempts the value of $\pm\left[-\frac{1}{4}x^4+\frac{8}{7}x^{\frac{7}{2}}+4x^{\frac{3}{2}}-14x^2-9x\right]_0^9$. No intermediate working is required.

This is dependent upon the first M and also obtaining '9x'

A1: AWRT 248. Allow the exact answer $\frac{6939}{28}$

Note that final answers of -248 and $|-248|$ are A0

Question Number	Scheme	Marks
8 (i)	$S_n = a + ar + ar^2 + ...ar^{n-1}$	B1
	$S_n = a + ar + ar^2 + ...ar^{n-1}$ **and** $rS = ar + ar^2 + ar^3 + ...ar^n$	
	Finds S_n and rS_n and subtracts. E.g. $S - rS = ...$	M1
	Completes proof $\Rightarrow S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1*
		(3)
(ii) (a)	Attempts $U = 150 \times 0.92^n$ with $n = 5 \text{ or } 6$	M1
	$\Rightarrow (U_6) = 150 \times (0.92)^6 = 90.95$ (≈ 91 litres) *	A1*
		(2)
(b)	Attempts $S_n = \dfrac{a(1-r^n)}{(1-r)}$ with $n = 39/40/41$ $a = 150/138$ and $r = 0.92$	M1
	$S = \dfrac{138(1-0.92^{40})}{(1-0.92)}$, $S = 150 \times \dfrac{0.92(1-0.92^{40})}{(1-0.92)}$ OR $S = \dfrac{150(1-0.92^{41})}{(1-0.92)} - 150$	A1
	1664 litres	A1
		(3)
		(8 marks)

Do not be too concerned with labelling here. It should be obvious which part of the question candidates are attempting to answer

(i) Condone $S \leftrightarrow S_n$ throughout.

A proof via the infinite sum formula gains no credit unless you see a correct expression for S_n.

B1: Scored for a correct expression for S_n. For example, $S_n = a + ar + ar^2 + ...ar^{n-1}$

including the "S_n" with a minimum of 3 terms and must include the first and the nth term.

$S_n = a + ar + ...ar^{n-1}$ would be acceptable but $S_n = a + ...ar^{n-1}$ is not.

It cannot be scored if some terms are incorrect $S_n = a + ... + \underline{\underline{ar^n}} + ar^{n-1}$

Or they are not in the correct order $S_n = a + ...ar^{n-1} + ar^{n-2}$

M1: States $S_n = a + ar + ar^2 + ...ar^{n-1}$ (minimum of 3 **correct** terms including **first** and **last** terms)

and $rS_n = ar + ar^2 + ...ar^n$ (minimum of 3 correct terms including first and last terms)

followed by an attempt to subtract either way around

A1*: Fully correct proof. Look for the following 3 lines (or their equivalents) following B1, M1

- $S_n - rS_n = a - ar^n$
- $S_n(1-r) = a(1-r^n)$
- $S_n = \dfrac{a(1-r^n)}{(1-r)}$ (Note that this is a given answer)