Question Number	Scheme	Marks
4. (a)	$\log_3(a+1)-\log_3 a=4 \Rightarrow \log_3\left(\frac{a+1}{a}\right)=4$	M1
	$\Rightarrow \left(\frac{a+1}{a}\right)=3^4$	A1
	$\Rightarrow a+1=81a \Rightarrow a=\frac{1}{80}$	dM1, A1
		(4)
(b)	$\text{Area} \approx \frac{1}{2}\left\{\log_3\left(\frac{2}{1}\right)+\log_3\left(\frac{6}{5}\right)+2\left(\log_3\left(\frac{3}{2}\right)+\log_3\left(\frac{4}{3}\right)+\log_3\left(\frac{5}{4}\right)\right)\right\}$	M1, A1
	$=\frac{1}{2}\log_3\left(\frac{2}{1}\times\frac{6}{5}\times\frac{3^2}{2^2}\times\frac{4^2}{3^2}\times\frac{5^2}{4^2}\right)=\frac{1}{2}\log_3 15=\log_3\sqrt{15}$	dM1, A1
		(4)
(c)	States 'increase the number of strips'	B1
		(1)
		(9 marks)

(a) Allow with $a \leftrightarrow x$ as seen in the notes

M1: Sets $\log_3(x+1)-\log_3 x=4$ and then applies the subtraction law. Condone the omission of the 3 in $\log_3$

Alternatively writes the equation $\log_3(x+1)=4+\log_3 x$ as $\log_3(x+1)=\log_3 k+\log_3 x \quad (k\neq 4)$ and then uses the addition law $\log_3(x+1)=\log_3(kx)$

A1: Correct equation not involving logs. For example, $\left(\frac{x+1}{x}\right)=3^4$

dM1: Correct attempt to solve an equation of the form $\frac{x+1}{x}=k$ o.e. where k is an integer $k\neq 3$ or 4.

Look for $\frac{x+1}{x}=k \Rightarrow x=\frac{1}{"k"-1}$ where $k\neq 3$.

A1: a or $x=\frac{1}{80}$ or 0.0125 following the award of M1,A1, dM1.

Working must be seen as required by the question.

Special Case:

You may see attempts that proceed towards the correct answer using incorrect laws.

E.g. $\log_3(x+1)-\log_3 x=4 \Rightarrow \frac{\log_3(x+1)}{\log_3 x}=4 \Rightarrow \left(\frac{x+1}{x}\right)=3^4 \Rightarrow x=\frac{1}{80}$

Candidates who achieve the correct answer using the above incorrect line can score SC 0, 0, 1, 1

(b)

Condone the omission of the 3 in $\log_3$ except for the final A mark where they must arrive at $\log_3 \sqrt{15}$.

Any use of calculators to produce decimal equivalents to these values will result in a maximum score of 1,0,0,0 marks as an exact value could never be achieved.

M1: Score for a reasonable attempt at applying the trapezium rule.

Look for **an attempt** at the correct form $\frac{h}{2}\{\text{first term} + \text{last term} + 2(\text{sum of the other terms})\}$ with at least the following

(1) Correct first **or** last term
(2) At least two correct 'other terms'
(3) $h = 1$

The bracketing must be correct but condone a missing trailing bracket.

So, look for a minimum of $\frac{1}{2}\{a + e + 2\times(b + c + d)$ with *a* or *e* correct, two out of three of *b*, *c* and *d* correct with the missing trailing bracket

FYI the decimal solution is $\frac{1}{2}\{0.631 + 0.166 + 2(0.369 + 0.262 + 0.203)\}$ allow awrt 2 dp

A1: A fully correct application of the trapezium rule in un-simplified form.

Amongst various correct alternatives are

$$\text{Area} \approx \frac{1}{2}\left\{\log_3\left(\frac{2}{1}\right) + \log_3\left(\frac{6}{5}\right) + 2\left(\log_3\left(\frac{3}{2}\right) + \log_3\left(\frac{4}{3}\right) + \log_3\left(\frac{5}{4}\right)\right)\right\}$$

$$\text{Area} \approx \frac{1}{2}\{\log_3 2 + \log_3 6 - \log_3 5 + 2(\log_3 3 - \log_3 2 + \log_3 4 - \log_3 3 + \log_3 5 - \log_3 4)\}$$

$$\text{Area} \approx \frac{1}{2}\{\log_3 2 + \log_3 6 - \log_3 5 + 2(-\log_3 2 + \log_3 5)\}$$

$$\text{Area} \approx \frac{1}{2}\left\{\log_3 2 + \log_3 \frac{3}{2}\right\} + \frac{1}{2}\left\{\log_3 \frac{3}{2} + \log_3 \frac{4}{3}\right\} + \frac{1}{2}\left\{\log_3 \frac{4}{3} + \log_3 \frac{5}{4}\right\} + \frac{1}{2}\left\{\log_3 \frac{5}{4} + \log_3 \frac{6}{5}\right\}$$

FYI: A fully simplified expression for the approximate area is $\frac{1}{2}\{\log_3 6 + \log_3 5 - \log_3 2\}$

Condone the omission of the trailing bracket as long as subsequent work implies its presence.

dM1: Correct use of logs in an attempt to fully combine their terms following the award of the previous M.

This must include sight of at least one correct use of two of the three log laws.

So, use of just addition and subtraction laws is fine as would addition and power laws

This mark can be implied if they go directly from a correct exact expression to the correct answer.

Candidates must proceed as far as $\frac{1}{2}\log p$ or $\log q$

A1: $\log_3 \sqrt{15}$ following M1, A1 dM1

(c)

B1: States

- 'increase the number of strips'
- 'decrease the width of the strips'
- 'make more strips'

without any incorrect statements such as increase the strips, so try $\int_1^{20} \log_3(x+1) - \log_3 x \, dx$

Do not allow vague statements such as 'add more numbers of *x*'

Question Number	Scheme	Marks
5. (a)	$u_2 = 1-\frac{1}{4}=\frac{3}{4}, \quad u_3 = 1-\frac{4}{3}=-\frac{1}{3},$	M1 A1
	$u_4 = 1-\left(-\frac{3}{1}\right)=4$ with statement saying the sequence repeats every 3 terms o.e	A1*
		(3)
(b)	$\sum_{n=1}^{180}(5n+3)=\frac{180}{2}(8+903)=(81990)$	M1
	$\sum_{n=1}^{180}(u_n)=60\times\left(4+\frac{3}{4}+-\frac{1}{3}\right)=(265)$	M1
	$\sum_{n=1}^{180}(5n+3+u_n)=81990+265=82255$	A1, A1
		(4)
		(7 marks)

(a)

M1: Attempts to use the iterative sequence correctly at least once.

Most often this is for $u_2 = 1-\frac{1}{4}$ or $u_2 = \frac{3}{4}$ o.e.

Condone omission of the lhs as long as it is obvious. E.g. 4, 0.75

A1: Achieves a correct value for both u_2 and u_3. Condone $u_3 = -0.\dot{3}$

Do not allow $u_3 = -0.33$ unless you see the exact value beforehand.

Allow $(4), \frac{3}{4}, -\frac{1}{3}$ if it is obviously values for both u_2 and u_3.

A1*: This is a show that question and requires

- correct values for u_2 and u_3
- a correct calculation and value seen for u_4 E.g. $u_4 = 1-\left(-\frac{3}{1}\right)=4$
- a minimal reason/statement 'the sequence repeats every 3 terms' o.e such as 'as $u_4 = u_1$,
- a minimal conclusion such as sequence is periodic of order 3' or 'hence true'