Question Number	Scheme	Marks
8 (i)	$3\tan\left(2x+\frac{\pi}{5}\right)=\sqrt{3}\Rightarrow 2x+\frac{\pi}{5}=\arctan\left(\frac{\sqrt{3}}{3}\right)$	M1
	$\Rightarrow 2x+\frac{\pi}{5}=\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}\Rightarrow x=...$	dM1
	$\Rightarrow x=\frac{29\pi}{60},\frac{59\pi}{60}$	A1
		(3)
(ii)	$5\sin\theta\tan\theta=\cos\theta+4$	
	$\tan\theta=\frac{\sin\theta}{\cos\theta}\rightarrow\quad 5\sin\theta\times\frac{\sin\theta}{\cos\theta}=\cos\theta+4$	M1
	$\Rightarrow 5\left(1-\cos^2\theta\right)=\cos^2\theta+4\cos\theta$	dM1
	$\Rightarrow 6\cos^2\theta+4\cos\theta-5=0$	A1
	$\Rightarrow\cos\theta=\frac{-4\pm\sqrt{16+120}}{12}$ **(**$=0.63849...$**)** $\Rightarrow\theta=$	ddM1
	$\Rightarrow\theta=50.3°, 309.7°$	A1
		(5)
		(8 marks)

In both (i) and (ii) condone poor notation, omission of θ or x in their working provided the intention is clear or recovered/implied by further work.

(i)

M1: Correct order of operations $2x+\frac{\pi}{5}=\arctan\left(\frac{\sqrt{3}}{3}\right)$ implied by $2x+\frac{\pi}{5}=\frac{\pi}{6}$ or any correct angle.

Condone decimal values here e.g. $2x+0.628...=0.52....$
Allow to work consistently in degrees. e.g. $2x+36=30$

Alternatively, attempts to use $\tan\theta=\frac{\sin\theta}{\cos\theta}$, squares both sides , attempts to use $\pm\sin^2\theta\pm\cos^2\theta=\pm1$ and proceeds to a quadratic equation in sine or cosine

e.g.

$$\frac{3\sin\left(2x+\frac{\pi}{5}\right)}{\cos\left(2x+\frac{\pi}{5}\right)}=\sqrt{3}\Rightarrow 3\sin\left(2x+\frac{\pi}{5}\right)=\sqrt{3}\cos\left(2x+\frac{\pi}{5}\right)\Rightarrow 9\sin^2\left(2x+\frac{\pi}{5}\right)=3\left(1-\sin^2\left(2x+\frac{\pi}{5}\right)\right)$$

$$\Rightarrow 12\sin^2\left(2x+\frac{\pi}{5}\right)=3\Rightarrow\sin\left(2x+\frac{\pi}{5}\right)=(\pm)\frac{1}{2}\Rightarrow 2x+\frac{\pi}{5}=\frac{\pi}{6},...$$

dM1: Correct order of operations $2x+\frac{\pi}{5}=\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}$ to find a value for x which does not need to be in the given range (e.g. condone $-\frac{\pi}{60}$)

Condone decimal answers here (awrt -0.05, awrt 1.5, awrt 3.1). It is dependent on the previous method mark. Allow to work consistently in degrees. e.g. $2x+36=30\Rightarrow x=\frac{30\pm 36}{2}=...$

A1: $(x=)\ \frac{29\pi}{60},\frac{59\pi}{60}$ and no other angles in the range. Must be exact and in radians.

Alternative: Using compound angle formula

M1: Forms the equation $\tan 2x=\dfrac{\frac{\sqrt{3}}{3}-\tan\left(\frac{\pi}{5}\right)}{1+\frac{\sqrt{3}}{3}\tan\left(\frac{\pi}{5}\right)}$ Allow to work consistently in degrees

dM1: Correct order of operations (arctan followed by dividing by 2)
Allow to work consistently in degrees

A1: $(x=)\ \frac{29\pi}{60},\frac{59\pi}{60}$ and no other angles in the range. Must be exact and in radians.

(ii) Note that there may be other credit worthy methods relying on identities covered in later units. If seen then send to review.

M1: Attempts to use $\tan\theta=\frac{\sin\theta}{\cos\theta}$ leading to an equation in $\sin\theta$ and $\cos\theta$

e.g. $5\sin\theta\times\frac{\sin\theta}{\cos\theta}=\cos\theta+4$ Condone slips.

dM1: Multiplies through by $\cos\theta$ (on at least 2 of the 3 terms in the equation) and attempts to use $\pm\sin^2\theta\pm\cos^2\theta=\pm1$ to set up a quadratic equation in $\cos\theta$ (may be in $\sin\theta$ if they make an earlier slip so allow this to score as well)
Condone slips e.g. not multiplying both terms by 5 if $5(1-\cos^2\theta)$ is not seen on the left hand side of the equation.
Terms do not need to be collected for this mark.
Condone arithmetical slips in their rearrangement.

A1: $6\cos^2\theta+4\cos\theta-5=0$ o.e The = 0 may be implied by further work.
Requires 3 terms but allow the constant term to be on the other side of the equation.

ddM1: Solves their **three-term** quadratic equation in $\cos\theta$ (or condone $\sin\theta$) and proceeds to a value for θ
Allow use of a calculator to solve the quadratic otherwise usual rules apply for factorising, using the formula or completing the square.
We must see

- a correct method used to solve the quadratic, or can be implied by a correct root to 1dp for their quadratic equation
- a correct angle for their root by taking arccos of the root (allow to the nearest integer in degrees) You may need to check this.

A correct angle on its own does not imply this mark.

A1: $(\theta=)$ awrt 50.3°, awrt 309.7° and no others in the range. Must be in degrees.

Question Number	Scheme	Marks
9 (a)	Attempts $12 = 3x^2 \times L \Rightarrow L = \dfrac{4}{x^2}$ or $Lx = \dfrac{4}{x}$	M1, A1
	$S = 6x^2 + 5xL = 6x^2 + \dfrac{20}{x}$	dM1, A1
		(4)
(b)	$\dfrac{\mathrm{d}S}{\mathrm{d}x} = 12x - \dfrac{20}{x^2}$	M1, A1ft
	$\dfrac{\mathrm{d}S}{\mathrm{d}x} = 0 \Rightarrow x^3 = \dfrac{5}{3} \Rightarrow x = 1.18563...$	dM1
	$S\big\vert_{x=1.18563} = 6 \times "1.18563"^2 + \dfrac{20}{"1.18563"} = 25.3\,(\mathrm{m}^2)$	ddM1, A1
		(5)
(c)	$\left(\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2} =\right) 12 + \dfrac{40}{x^3}$	M1
	Justifies $\dfrac{\mathrm{d}^2S}{\mathrm{d}x^2} > 0$ (at $x = 1.18563...$), hence minimum *	A1*
		(2)
		(11 marks)

(a)

M1: Attempts to use a volume formula to make L or Lx the subject. May be seen substituted into a surface area expression which is not in the required form of the final answer.

Allow $12 = \alpha x^2 L \Rightarrow L = \dfrac{\beta}{x^2}$ or $Lx = \dfrac{\beta}{x}$

A1: $L = \dfrac{4}{x^2}$ OR $Lx = \dfrac{4}{x}$ o.e. Maybe seen substituted into the equation for the surface area.

dM1: Substitutes $L = \dfrac{"4"}{x^2}$ OR $Lx = \dfrac{"4"}{x}$ into $S = Rx^2 + TxL$ which may be unsimplified to produce a formula of the correct form. It is dependent on the previous method mark.

A1: $S = 6x^2 + \dfrac{20}{x}$

Must see "$S =$" or e.g. "(surface) area = " connected at some point with $6x^2 + \dfrac{20}{x}$