Question Number	Scheme		Marks
	OPEN TOPPED	**CLOSED CONTAINER**	
3.(a)	$120=3x^2y \Rightarrow y=\dfrac{40}{x^2}$	$120=3x^2y \Rightarrow y=\dfrac{40}{x^2}$	M1, A1
	$A=3x^2+8xy$	$A=6x^2+8xy$	
	$=3x^2+8x\times\dfrac{40}{x^2}=3x^2+\dfrac{320}{x}$	$=6x^2+8x\times\dfrac{40}{x^2}=6x^2+\dfrac{320}{x}$	dM1, A1
			(4)
(b)	$\dfrac{\mathrm{d}A}{\mathrm{d}x}=6x-\dfrac{320}{x^2}$	$\dfrac{\mathrm{d}A}{\mathrm{d}x}=12x-\dfrac{320}{x^2}$	M1, A1ft
	$\dfrac{\mathrm{d}A}{\mathrm{d}x}=6x-\dfrac{320}{x^2}=0$	$\dfrac{\mathrm{d}A}{\mathrm{d}x}=12x-\dfrac{320}{x^2}=0$	
	$\Rightarrow x^3=\dfrac{160}{3}\Rightarrow x=$ awrt 3.76	$\Rightarrow x^3=\dfrac{80}{3}\Rightarrow x=$ awrt 2.99	dM1, A1
			(4)
(c)	Attempts $\left.\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}\right\|_{x=3.76}=6+\dfrac{640}{x^3}$	Attempts $\left.\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}\right\|_{x=2.99}=12+\dfrac{640}{x^3}$	M1
	$=6+\dfrac{640}{3.76^3}=..$	$=12+\dfrac{640}{2.99^3}=..$	
	$\left.\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}\right\|_{x=3.76}=$ awrt $18>0$ MINIMUM*	$\left.\dfrac{\mathrm{d}^2A}{\mathrm{d}x^2}\right\|_{x=2.99}=$ awrt $36>0$ MINIMUM*	A1*
			(2)
			(10 marks)

Note that candidates may be confused regarding open topped or closed container so both cases can potentially score full marks if the case is consistently applied.

(a)

M1: Sets $120=k\,x^2y$ o.e such as $120=k\,x\times x\times y$

A1: $y=\dfrac{40}{x^2}$ or $xy=\dfrac{40}{x}$ or equivalent such as $y=\dfrac{120}{3x^2}$

It can be scored from an embedded expression within an attempt at A.

It can be implied following a correct $120=3x^2y$ o.e. and a correct expression for A in terms of x

dM1: Achieves an expression of the form $(A=)\,px^2+qxy,\ p,q>0$ **AND** substitutes in their y or xy of the form $y=\dfrac{K}{x^2}$ or $xy=\dfrac{K}{x}$ which may be implied by a correct expression for A in terms of x

A1: Achieves the answer of Area / $A=3x^2+\dfrac{320}{x}$ **or** Area/ $A=6x^2+\dfrac{320}{x}$

Open Container *Closed Container*

The LHS A = or an equivalent such as 'Area =' must appear at some point.
Condone A appearing, for example, as SA or S or Area (of sheet metal) or other but a LHS must be seen.

Mark parts (b) and (c) together.

(b) Condone $\frac{dA}{dx}$ appearing as $\frac{dy}{dx}$, y' or A' or the absence of a LHS **throughout part (b)**

M1: $\frac{dA}{dx} = \alpha x \pm \frac{\beta}{x^2}$.

Allow this mark to be scored with the constants P and Q or for made up values of P and Q

A1ft: $\frac{dA}{dx} = 6x - \frac{320}{x^2}$ o.e. or $\frac{dA}{dx} = 12x - \frac{320}{x^2}$ following through on their numerical P and Q

Open Container *Closed Container*

It can be scored for made up values of P and Q

dM1: Sets $\frac{dA}{dx} = \alpha x - \frac{\beta}{x^2} = 0$ and proceeds to a numerical $x^3 = k$ $k > 0$ but allow $\alpha x^3 = \beta$ $\alpha\beta > 0$

Condone $\frac{dA}{dx} = \alpha x - \frac{\beta}{x^2} = 0$ leading to a correct value for x for their $\frac{dA}{dx} = 0$

A1: $x =$ awrt 3.76 or $x =$ awrt 2.99

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(c) Condone $\frac{d^2A}{dx^2}$ being written as A'' throughout part (c)

M1: Achieves $\frac{d^2A}{dx^2} = \delta \pm \frac{\varepsilon}{x^3}$ and

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- Either attempts to find its value at their 3.76 or 2.99
- Or states it is positive (even following an incorrect value)

Condone $\frac{d^2A}{dx^2}$ as $\frac{d^2y}{dx^2}$ for this mark only

Allow this mark to be scored with the constants P and Q or for made up values of P and Q

A1*: Scored for

- A correct $\frac{d^2A}{dx^2}$ including the LHS. So look for $\frac{d^2A}{dx^2} = 6 + \frac{640}{x^3}$ or $\frac{d^2A}{dx^2} = 12 + \frac{640}{x^3}$

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- A correct statement and conclusion

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Either $\left.\frac{d^2A}{dx^2}\right|_{x=3.76} =$ awrt $18 > 0$ and MINIMUM $\left.\frac{d^2A}{dx^2}\right|_{x=2.99} =$ awrt $36 > 0$ and MINIMUM

Or $\frac{d^2A}{dx^2} = 6 + \frac{640}{x^3} > 0$ as $x > 0$ MINIMUM $\frac{d^2A}{dx^2} = 12 + \frac{640}{x^3} > 0$ as $x > 0$ MINIMUM

Or $\frac{d^2A}{dx^2} = 6 + \frac{640}{x^3} > 0$ when $x = 3.76$, MINIMUM $\frac{d^2A}{dx^2} = 12 + \frac{640}{x^3} > 0$ when $x = 2.99$

MINIMUM

Open Container *Closed Container*

The mark can only be awarded following $x = \text{awrt } 3.76$ or $x = \text{awrt } 2.99$

Condone statements like '$x = 3.76$ is the minimum value of A as $\frac{\mathrm{d}^2 A}{\mathrm{d}x^2} > 0$'.

Look for the key aspects '$\frac{\mathrm{d}^2 A}{\mathrm{d}x^2} > 0$' and 'minimum'. If in doubt use review

Question Number	Scheme	Marks
4 (a)	$\dfrac{a\left(1-r^3\right)}{1-r}=70.2$, $\dfrac{a}{1-r}=75$	B1, B1
	Sub (2) in (1) $\quad 75\left(1-r^3\right)=70.2 \Rightarrow r^3=\left(0.064\right)$	M1
	$r=0.4$	A1
		(4)
(b)	Substitutes $r=0.4$ into $\dfrac{a}{1-r}=75$ and finds a	M1
	$a=45$	A1
		(2)
		(6 marks)

(a)

B1: One correct equation. Most of the time it is one of

- $\dfrac{a}{1-r}=75$
- $\dfrac{a\left(1-r^3\right)}{1-r}=70.2$, $a+ar+ar^2=70.2$ o.e.

B1: Two different equations that allow the problem to be solved.

Another more complicated option that can be used (with either of the above bullets) is

$\dfrac{a}{1-r}-\dfrac{a\left(1-r^3\right)}{1-r}=4.8$

M1: Combines both necessary equations (condoning numerical slips e.g. 72.0 for 70.2) and proceeds to an equation of the form $\delta r^3=\beta$. Also condone the equations of the same form with sign slips

$\dfrac{a}{r-1}=75$

Working is necessary here so candidates cannot just write down the equations and the solution without sight of the line $\delta r^3=\beta$ or $\delta r^3-\beta=0$.

A more complicated method involves substituting $a=75-75r$ into $a-ar^3=70.2(1-r)$ to produce a simplified quartic equation in *r*. FYI the correct simplified equation is $75r^4-75r^3-4.8r+4.8\left(=0\right)$.

If they achieve a simplified quartic equation in *r* and write down a value for r $\left(r\neq 1, r\in \mathbb{R}\right)$ that solves their equation they can score M1. So, if they write down the correct simplified equation followed by $r=0.4$ from their calculator they can score M1, A1.

If you spot a method that does not proceed via one of these two methods and you feel deserves credit, then please send to review

A1: $r=0.4$ only, following award of the method mark.

(b)

M1: Substitutes their value of $r, \left|r\right|<1$, found in part (a) into one of their correct equations and find a value for *a*.

Candidates cannot just make up a value. It must follow a value in *r* found from an equation in just *r*.

A1: $a=45$ Can only be awarded if two correct equations are written down and $r=0.4$