(b)	$\int\left\{13-\frac{9}{x^2}-\left(x^2+3\right)\right\}\mathrm{d}x=13x+\frac{9}{x}-\frac{x^3}{3}-3x\left(+c\right)$ or $\int\left(13-\frac{9}{x^2}\right)\mathrm{d}x=13x+\frac{9}{x}\left(+c\right),\ \int\left(x^2+3\right)\mathrm{d}x=\frac{x^3}{3}+3x\left(+c\right)$	M1A1
	$\left[10x+\frac{9}{x}-\frac{x^3}{3}\right]_1^3=10\left(3\right)+\frac{9}{3}-\frac{3^3}{3}-\left(10+9-\frac{1}{3}\right)=...$ or $\left[13x+\frac{9}{x}\right]_1^3-\left[\frac{x^3}{3}+3x\right]_1^3=39+3-\left(13+9\right)-\left\{9+9-\left(\frac{1}{3}+3\right)\right\}=...$	**d**M1
	$=\frac{16}{3}$	A1
		(4)
		Total 8

Note that this question has the calculator warning so attempts that do not use algebraic integration score no marks.

(b)

M1: Evidence of integration $x^n \to x^{n+1}$ for at least one term for either **curve** or for the difference between the **2 curves** e.g. it is not for integrating x^4-10x^2+9

A1: Correct integration for both curves either as a difference (either way round) or separately. E.g. $\pm\left(13x+\frac{9}{x}-\frac{x^3}{3}-3x\left(+c\right)\right)$ or $13x+\frac{9}{x}$ and $\frac{x^3}{3}+3x$

Allow simplified or unsimplified but indices must be processed.

dM1: **Depends on the first method mark.**

Substitutes their positive values from part (a) and subtracts if attempted as a difference or substitutes their values into the separate integrations, subtracts and then subtracts the 2 results either way round to obtain a value.

Condone poor bracketing as long as the intention is clear.

May be implied by the correct final answer following correct integration.

If the integration and/or limits are incorrect **and** the substitution is not shown explicitly **then** you may need to check.

A1: $\frac{16}{3}$ or **exact** equivalent. $-\frac{16}{3}$ scores A0 unless it gets corrected to $+\frac{16}{3}$

Question Number	Scheme	Marks
9(a)	$\left(2\tan\theta = 3\cos\theta \Rightarrow\right) \dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta$	M1
	$\dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta \Rightarrow 2\sin\theta = 3\cos^2\theta = 3\left(1-\sin^2\theta\right)$	M1
	$2\sin\theta = 3\left(1-\sin^2\theta\right) \Rightarrow 3\sin^2\theta + 2\sin\theta - 3 = 0$*	A1*
		(3)
(b)	$\left(\sin\left(2x+\dfrac{\pi}{3}\right) =\right) \dfrac{-1\pm\sqrt{10}}{3}$ (May only see positive root) NB decimal roots are: $-1.387\ldots$, $0.7207\ldots$	M1
	$2x+\dfrac{\pi}{3} = \sin^{-1}(0.7207...) \Rightarrow x = ...$	M1
	$-0.121, -2.50, 0.645, 3.02$	A1A1
		(4)
		Total 7

(a)

For full marks condone a complete proof <u>entirely</u> in *x* (or another variable) instead of θ

M1: Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to write the equation in terms of sine and cosine only.

Must be the correct identity so e.g. $2\tan\theta = \dfrac{2\sin\theta}{2\cos\theta}$ is M0

May be implied by e.g. $2\tan\theta = 3\cos\theta \Rightarrow 2\sin\theta = 3\cos^2\theta$

M1: Uses $\cos^2\theta = 1-\sin^2\theta$ to obtain a quadratic equation in sine only.

Must be the correct identity so e.g. $3\cos^2\theta = 1-3\sin^2\theta$ is M0

A1*: Correct work with all necessary steps shown leading to the given answer.

Condone e.g. $2\sin\theta + 3\sin^2\theta - 3 = 0$

There should be **no notational or bracketing errors and no mixed or missing variables**. E.g. we would consider

- $\cos^2\theta$ written as $\cos\theta^2$ a notational error
- $\tan\theta = \dfrac{\sin}{\cos}\theta$ as a missing variable

<u>Working backwards:</u>

$$\left(3\sin^2\theta + 2\sin\theta - 3 = 0 \Rightarrow\right) 3\left(1-\cos^2\theta\right) + 2\sin\theta - 3 = 0$$

$$\Rightarrow 2\sin\theta - 3\cos^2\theta = 0$$

$$\Rightarrow 2\sin\theta = 3\cos^2\theta$$

$$\Rightarrow 2\tan\theta = 3\cos\theta$$* Hence proven

M1: Uses $\sin^2\theta = 1-\cos^2\theta$ to obtain an equation in sine and cosine^2 only.

M1: Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ which may be implied.

A1*: Reaches $2\tan\theta = 3\cos\theta$ with the same conditions as above but with a (minimal) conclusion.

(b)