(c)	$f(x)=(x-7)(2x^2+...x+...)+...$	M1
	$2x^2+13x+65$	A1
		(2)
		Total 7

(c) Do not allow mis-reads of (x – 7) in part (c).

M1: Uses any appropriate method e.g. long division/inspection/table with numeric non-zero A and B to obtain $2x^2 + px + q$ where p and q are non-zero.
There is no requirement to find the remainder but assuming (x – 7) is a **factor** is M0 (see example below)

A1: Must follow correct values of *A* and *B*.
Correct expression $2x^2+13x+65$ **which is clearly identified** e.g. by circling/underlining or e.g. Quotient = $2x^2+13x+65$, $Q=2x^2+13x+65$
Any remainder, correct or incorrect, can be ignored. (NB correct remainder is 495)

Long division for reference:

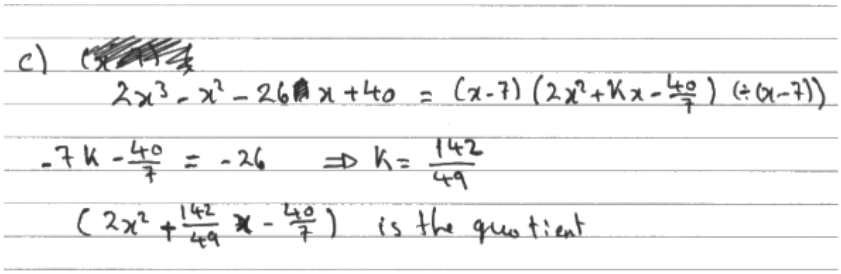
$$\begin{array}{r} 2x^2+13x+65 \\ x-7\overline{)\,2x^3-x^2-26x+40} \\ \underline{2x^3-14x^2} \\ 13x^2-26x+40 \\ \underline{13x^2-91x} \\ 65x+40 \\ \underline{65x-455} \\ 495 \end{array}$$

Tabular method for reference:

	$2x^2$	$13x$	65
x	$2x^3$	$13x^2$	$65x$
–7	$-14x^2$	$-91x$	–455

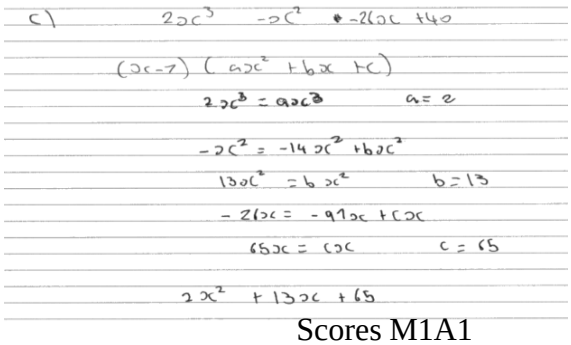
But note that **just** the above score M1A0 with no indication that the quotient is $2x^2+13x+65$

Some candidates assume that (x – 7) is a **factor** of f(x) and try to find the quadratic factor:



This scores M0 in part (c)

But work that doesn't assume (x – 7) is a factor is fine e.g.:



Scores M1A1

Question Number	Scheme	Marks
4(a)	$y = 4x^{\frac{1}{2}} + 9x^{-\frac{1}{2}} + 3$ $\Rightarrow \left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}}$	M1A1
		(2)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow 4x - 9 = 0 \Rightarrow x = ...$	M1
	$x = \frac{9}{4}$ oe e.g. 2.25	A1
		(2)

(a)

M1: For $x^{\frac{1}{2}} \to x^{-\frac{1}{2}}$ or $x^{-\frac{1}{2}} \to x^{-\frac{3}{2}}$. Allow unprocessed e.g. $x^{\frac{1}{2}} \to x^{\frac{1}{2}-1}$ or $x^{-\frac{1}{2}} \to x^{-\frac{1}{2}-1}$.

A1: Correct simplified derivative. $\frac{\mathrm{d}y}{\mathrm{d}x} =$ is not required. Isw once a correct answer is seen.

Allow equivalent simplified expressions e.g. $2x^{-\frac{1}{2}} - 4.5x^{-\frac{3}{2}}$

$2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} + c$ scores A0 as does $2x^{-\frac{1}{2}} + -\frac{9}{2}x^{-\frac{3}{2}}$

(b)

M1: Starts from a derivative of the form $Ax^{-\frac{1}{2}} - Bx^{-\frac{3}{2}}$ where $A \times B > 0,$ sets = 0 (seen or implied) and solves via a correct method to obtain an expression or value of $x = \frac{B}{A}$.

A1: $x = \frac{9}{4}$ oe with no incorrect working seen in their method.

Condone if e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} > 0$ is seen as long as they reach the correct value for *x*.

There must be no other values. **Must come from a correct derivative but see * note below.**

The minimum acceptable for both marks is $2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow x = \frac{9}{4}$

The following is condoned:

$$2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 0 \Rightarrow \frac{2}{x^{\frac{1}{2}}} - \frac{9}{2x^{\frac{3}{2}}} = 0 \Rightarrow \frac{4}{x} - \frac{81}{4x^3} = 0 \Rightarrow x = \frac{9}{4}$$

Ignore any attempts to find the *y* coordinate.

***Allow full recovery in (b) if the working is correct in (a) but isw has been applied e.g.**

(a) $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right) 2x^{-\frac{1}{2}} - \frac{9}{2}x^{-\frac{3}{2}} = 4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}}$ M1A1 (isw)

(b) $4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}} = 0 \Rightarrow x = \frac{9}{4}$ M1A1

4(c)(i)	$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\right)-x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$ oe e.g. $-x^{-\frac{3}{2}}+6.75x^{-\frac{5}{2}}$	B1ft
(ii)	$\left(\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_{x=\frac{9}{4}}=\right)-\left(\frac{9}{4}\right)^{-\frac{3}{2}}+\frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}}\left(=\frac{16}{27}(0.5925...)\right)$ $\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)>0$ so (local) minimum	B1
		(2)
(d)	$0<x<\frac{9}{4}$	B1ft
		(1)
		Total 7

(c)(i)

B1ft: Correct simplified second derivative.
Follow through their first derivative provided there are 2 terms with **different** fractional (non-integer) powers.

$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=$ is not required.

Condone $-1x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$ for $-x^{-\frac{3}{2}}+\frac{27}{4}x^{-\frac{5}{2}}$

Inclusion of "+ *c*" scores B0ft as with the first derivative.

(ii)

B1: Fully correct working, reasoning and conclusion. This requires:

- a correct second derivative
- substitutes $x=\frac{9}{4}$ to obtain $\frac{16}{27}$ or awrt 0.6 or truncated 0.5
- reference to the sign (> 0 or positive). Must see the evaluation as it is not obviously positive but allow if expression is not fully evaluated as long as it is clear it is positive (e.g. $\frac{8}{9}-\frac{8}{27}>0$).
- reference to minimum

Condone work that doesn't specifically reference $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ e.g.

$-\left(\frac{9}{4}\right)^{-\frac{3}{2}}+\frac{27}{4}\left(\frac{9}{4}\right)^{-\frac{5}{2}}=\frac{16}{27}>0$ so minimum is fine for this mark.

(d)

B1ft: For $0<x<\frac{9}{4}$. Allow $0<x$,, $\frac{9}{4}$ and allow equivalent statements e.g.

$\left(0,\frac{9}{4}\right)$, $\left(0,\frac{9}{4}\right]$.

Follow through their positive *x* value from part (b). Condone an answer of $x<\frac{9}{4}$ or

x ,, $\frac{9}{4}$ Condone 0 ,, x ,, $\frac{9}{4}$

Do not allow $-\infty<x<\frac{9}{4}$