Question Number	Scheme	Marks
10(a)	$x^2+y^2+4x-30y+209=0$ $\Rightarrow (x\pm 2)^2+(y\pm 15)^2 ...=0$	M1
(i)	Centre (–2, 15)	A1
(ii)	Radius $\sqrt{20}$	A1
		(3)
(b)	$y=mx+1 \Rightarrow (x+2)^2+(mx+1-15)^2=20$ or $y=mx+1 \Rightarrow x^2+(mx+1)^2+4x-30(mx+1)+209=0$	M1
	$x^2+m^2x^2+4x-28mx+180=0$ $b^2-4ac=0 \Rightarrow (4-28m)^2-4(1+m^2)\times 180=0$	**d**M1
	$(4-28m)^2-4(1+m^2)180=0 \Rightarrow 16-224m+784m^2-720-720m^2=0$ $\Rightarrow 2m^2-7m-22=0$*	A1*
		(3)
(c)	$2m^2-7m-22=0 \Rightarrow m=\frac{11}{2}, -2$	M1
	$m=\frac{11}{2} \Rightarrow \frac{125}{4}x^2-150x+180=0 \Rightarrow x=\frac{12}{5} \Rightarrow y=\frac{71}{5}$ or $m=-2 \Rightarrow 5x^2+60x+180=0 \Rightarrow x=-6 \Rightarrow y=13$	M1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ **or** $(-6, 13)$ oe	A1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ **and** $(-6, 13)$ oe	A1
		(4)
		Total 10

(a) Look out for answers to part (a) written in the body of the question.

M1: Attempts to complete the square for **both** variables or states a centre of (±2, ±15)
For completing the square allow $(x\pm 2)^2 ...(y\pm 15)^2 ...=...$

A1: Centre (–2, 15) or e.g. $x=-2$, $y=15$. Condone e.g. –2, 15 or [–2, 15]

A1: For $\sqrt{20}$ or $2\sqrt{5}$ which may be scored following $(x\pm 2)^2+(y\pm 15)^2=20$
Do **not** allow $\pm\sqrt{20}$ but apply isw if a correct radius is seen which is then e.g. converted to a decimal or simplified incorrectly.

(b) Mark (b) and (c) together.

M1: Attempts to substitute $y = mx + 1$ into the given equation or their rearranged equation to obtain an equation in m and x only. Condone slips as long as the intention is clear.

dM1: Attempts $b^2-4ac...0$ where … is "=" or ">" or "<" etc. or equivalent e.g. $b^2=4ac$ where $a=A+Bm^2$, $b=C+Dm$, $c=E$ where A, B, C, D and E are non-zero constants. Condone copying slips if the intention is clear if the above conditions are met. **Depends on the first M mark.**

A1*: Obtains the printed answer with no errors e.g. must have had $b^2-4ac=0$ or equivalent throughout and before the final line e.g. not $b^2-4ac>0$ and brackets expanded. **Note this is a given answer so must follow correct work.**

(b) Alternative: Perpendicular distance from a point to a line:

$$(-2, 15),\quad mx - y + 1 = 0 \quad \text{with } d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

$$\sqrt{20} = \frac{|-2m - 15 + 1|}{\sqrt{m^2 + 1}} \Rightarrow 20\left(m^2 + 1\right) = \left(2m + 14\right)^2$$

$$\Rightarrow 16m^2 - 56m - 176 = 0 \Rightarrow 2m^2 - 7m - 22 = 0*$$

M1: Substitutes into a correct distance formula with their centre and the given L and sets equal to their radius.

dM1: Squares, multiplies up to obtain $\alpha\left(Am^2 + B\right) = \beta\left(Cm + D\right)^2$ oe, where A, B, C and D are non-zero.

A1*: Obtains the printed answer with no errors

(c)

M1: Solves the **given** quadratic equation by any valid means, including calculator, to obtain at least one value for m. May be implied by their value(s).
One correct value **only** with no incorrect work is sufficient for M1
Condone $x = \ldots$ for this mark.

M1: Uses at least one of their values of m to attempt one position for P.
This must be a **complete and correct method to find a position for *P*** i.e. finds at least one value for x **and** then the corresponding value for y correctly.
E.g. substitutes at least one value for x into their $\left(x + 2\right)^2 + \left(mx + 1 - 15\right)^2 = 20,$ solves the resulting 3TQ by any method including a calculator and then uses the correct $y = mx + 1$ to find the y value. Condone slips when e.g. simplifying their 3TQ providing the complete method is correct.

A1: At least one correct point. Allow as a coordinate pair or as $x = \ldots$, $y = \ldots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \ldots$, $y = \ldots$

(c) Alternative 1:

Finds the intersections of the possible tangents with the perpendiculars passing through the centre of the circle:

M1: $\left(m=\frac{11}{2},\ -2\right)$ As above

$$\text{For } m=\frac{11}{2} \text{ perpendicular is } y-15=-\frac{2}{11}(x+2)\left(y=-\frac{2}{11}x+\frac{161}{11}\right)$$

$$-\frac{2}{11}x+\frac{161}{11}=\frac{11}{2}x+1\Rightarrow x=\frac{12}{5},\ y=\frac{71}{5}$$

or

$$\text{For } m=-2 \text{ perpendicular is } y-15=\frac{1}{2}(x+2)\left(y=\frac{1}{2}x+16\right)$$

$$\frac{1}{2}x+16=-2x+1\Rightarrow x=-6,\ y=13$$

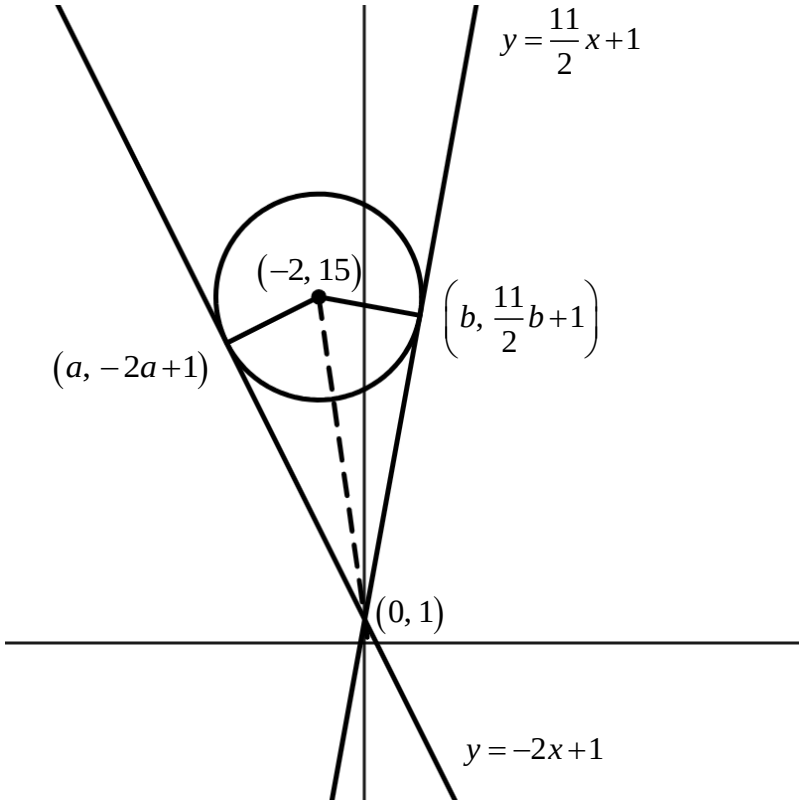
M1: For a complete method by

- forming the equation of at least one of the perpendiculars, with the negative reciprocal gradient and the coordinates of their centre correctly placed
- solving simultaneously with the corresponding tangent to find x or y
- finding the corresponding x or y coordinate

A1: At least one correct point. Allow as a coordinate pair or as $x = \ldots$, $y = \ldots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \ldots$, $y = \ldots$

(c) Alternative 2:



$$(15-1)^2+2^2=a^2+(2a)^2+\left(\sqrt{20}\right)^2$$

or

$$(15-1)^2+2^2=b^2+\left(\frac{11}{2}b\right)^2+\left(\sqrt{20}\right)^2$$

Then

$$a^2=36\Rightarrow a=-6\Rightarrow -2a+1=13$$

or

$$b^2=\frac{144}{25}\Rightarrow b=\frac{12}{5}\Rightarrow \frac{11}{2}b+1=\frac{71}{5}$$

Score as:

M1: As above

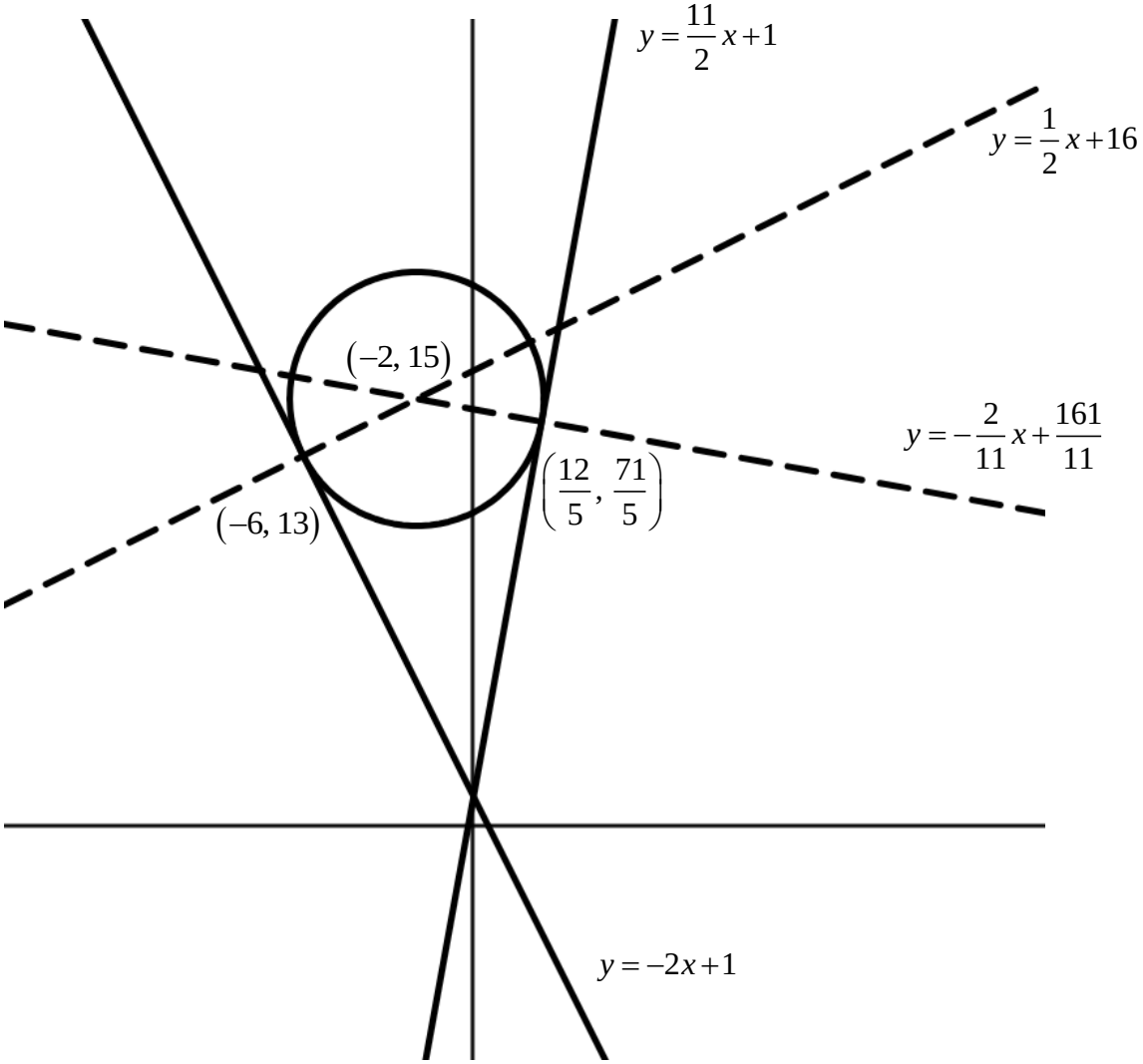
M1: For a complete method by

- using Pythagoras correctly to find the distance or distance^2 between (0, 1) and their centre
- applying Pythagoras correctly with a general point on either tangent and their radius to obtain an equation in one variable and solves
- finding the corresponding x or y coordinate

A1: At least one correct point. Allow as a coordinate pair or as $x = \ldots$, $y = \ldots$ and allow equivalent values e.g. (2.4, 14.2) etc.

A1: Both points correct and no others. Allow as coordinate pairs or as $x = \ldots$, $y = \ldots$

General diagram for reference:



Question Number	Scheme	Marks
11(i)	E.g. $n = 5 \Rightarrow 3^5 + 2 = 245$	M1
	245 is not a prime number or e.g. 245 is divisible by 5 so not true	A1
		(2)

M1: Attempts to evaluate $3^n + 2$ with n a prime number.
May be seen as a list of attempts with various numbers even if some are non-prime e.g. $n = 1$ as long as **at least one prime number** is attempted.
Condone a slip in evaluating provided the intention was to substitute in a valid value for n.

A1: A correct calculation for the value and a conclusion.
There must be some reference to it not being prime **or** they show that the number is divisible by e.g. 5 and state that it is false/not true.
The value they have chosen must be clearly identified as their counter example and other considerations of n can be ignored.

Examples:

$3^5 + 2 = 245$ which is not prime is M1A1

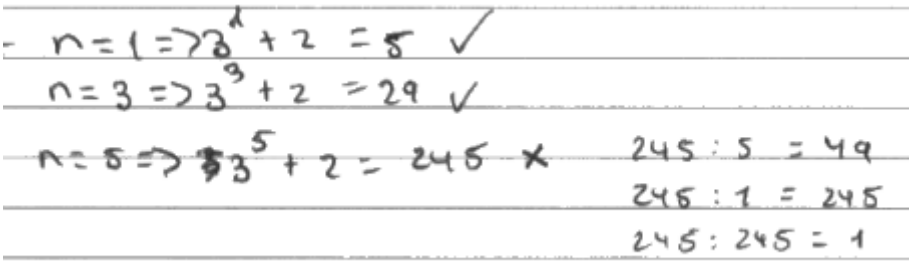
when $n = 11$ then $3^n + 2 = 177149$ and $177149 \div 7 = 25307$ so false is M1A1

$3^5 + 2 = 245$ so false is M1A0

$3^5 + 2 = 245$ which is divisible by 25 so not prime is M1A0

Most candidates are likely to choose $n = 5$ as their counter example but other numbers are possible e.g.:

n	$3^n + 2$
7	2189
11	177149
13	1594325
17	129140165



Scores: M1A0 (we would need e.g. "not prime" or 245/5 = 49 so false) but note that we would have ignored the consideration of any non-primes e.g. $n = 1$

(ii)	$m=3k+1 \Rightarrow m^2-1=(3k+1)^2-1=9k^2+6k+1-1$ **or** $m=3k+2 \Rightarrow m^2-1=(3k+2)^2-1=9k^2+12k+4-1$	M1
	$m^2-1=9k^2+6k=3(3k^2+2k)$ **or** $m^2-1=9k^2+12k+3=3(3k^2+4k+1)$	A1
	$m=3k+1 \Rightarrow m^2-1=(3k+1)^2-1=9k^2+6k+1-1$ **and** $m=3k+2 \Rightarrow m^2-1=(3k+2)^2-1=9k^2+12k+4-1$	**d**M1
	$3(3k^2+2k)$ and $3(3k^2+4k+1)$ are both multiples of 3 so m^2-1 must be divisible by 3 when *m* is not divisible by 3	A1
		(4)
		Total 6

For the accuracy marks there should be no errors in the algebra but allow e.g. invisible brackets to be "recovered".
Withhold the final mark if *m* is used instead of their *k* if the work is otherwise correct.

M1: Starts the proof by considering at least one algebraic expression not divisible by 3 e.g. $m = 3k + 1$ and attempts m^2-1 by expanding the brackets.
Alternatively, writes $m^2-1=(m-1)(m+1)$ and substitutes at least one algebraic expression not divisible by 3.
Allow equivalent representations of numbers not divisible by 3 e.g. $m = 3k + 2$ or $m = 3k - 1$
Condone arithmetical slips and condone the use of e.g. $m = 3m + 2$ or $m=3m-1$

A1: They must

- obtain at least one correct expression
- show that their expression is a multiple of 3 by factoring out 3 or e.g. $3k$ e.g. $3(3k^2+2k)$ or e.g. $3k(3k+2)$ or e.g. $3(k+1)(3k+1)$

  or

  show that their expression is divisible by 3 e.g. $\frac{9k^2+6k}{3}=3k^2+2k$.

  or

  make a reasoned argument as to why the expression is divisible by 3 e.g. obtains $9k^2+6k$ and states e.g. is divisible by 3 as both 9 and 6 are divisible by 3 (or equivalent reasoning)
  Note that this may be seen in the final conclusion.

Condone a spurious "= 0" e.g. $3(3k^2+2k)=0$ **for this mark**.

dM1: Considers both **distinct** cases where *m* is not divisible by 3 (see above)