Question Number	Scheme	Marks
9(a)	$\left(2\tan\theta = 3\cos\theta \Rightarrow\right) \dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta$	M1
	$\dfrac{2\sin\theta}{\cos\theta} = 3\cos\theta \Rightarrow 2\sin\theta = 3\cos^2\theta = 3\left(1-\sin^2\theta\right)$	M1
	$2\sin\theta = 3\left(1-\sin^2\theta\right) \Rightarrow 3\sin^2\theta + 2\sin\theta - 3 = 0$*	A1*
		(3)
(b)	$\left(\sin\left(2x+\dfrac{\pi}{3}\right) =\right) \dfrac{-1\pm\sqrt{10}}{3}$ (May only see positive root) NB decimal roots are: $-1.387\ldots$, $0.7207\ldots$	M1
	$2x+\dfrac{\pi}{3} = \sin^{-1}(0.7207...) \Rightarrow x = ...$	M1
	$-0.121, -2.50, 0.645, 3.02$	A1A1
		(4)
		Total 7

(a)

For full marks condone a complete proof <u>entirely</u> in *x* (or another variable) instead of θ

M1: Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to write the equation in terms of sine and cosine only.

Must be the correct identity so e.g. $2\tan\theta = \dfrac{2\sin\theta}{2\cos\theta}$ is M0

May be implied by e.g. $2\tan\theta = 3\cos\theta \Rightarrow 2\sin\theta = 3\cos^2\theta$

M1: Uses $\cos^2\theta = 1-\sin^2\theta$ to obtain a quadratic equation in sine only.

Must be the correct identity so e.g. $3\cos^2\theta = 1-3\sin^2\theta$ is M0

A1*: Correct work with all necessary steps shown leading to the given answer.

Condone e.g. $2\sin\theta + 3\sin^2\theta - 3 = 0$

There should be **no notational or bracketing errors and no mixed or missing variables**. E.g. we would consider

- $\cos^2\theta$ written as $\cos\theta^2$ a notational error
- $\tan\theta = \dfrac{\sin}{\cos}\theta$ as a missing variable

<u>Working backwards:</u>

$$\left(3\sin^2\theta + 2\sin\theta - 3 = 0 \Rightarrow\right) 3\left(1-\cos^2\theta\right) + 2\sin\theta - 3 = 0$$

$$\Rightarrow 2\sin\theta - 3\cos^2\theta = 0$$

$$\Rightarrow 2\sin\theta = 3\cos^2\theta$$

$$\Rightarrow 2\tan\theta = 3\cos\theta$$* Hence proven

M1: Uses $\sin^2\theta = 1-\cos^2\theta$ to obtain an equation in sine and cosine^2 only.

M1: Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ which may be implied.

A1*: Reaches $2\tan\theta = 3\cos\theta$ with the same conditions as above but with a (minimal) conclusion.

(b)

M1: Attempts to solve the quadratic $3\sin^2 x + 2\sin x - 3 = 0$ to obtain a value for sin *x* where *x* is any variable. Usual rules apply for solving a quadratic (via a calculator is also acceptable and may imply this mark). If no working is shown then the root(s) must be correct but condone premature rounding e.g. 0.72, -1.3

M1: Attempts to find one angle within the range by finding the inverse sine of one of their roots, subtracting $\frac{\pi}{3}$ and dividing by 2.

For this mark allow to work in degrees if done correctly. E.g. they would need to change $\frac{\pi}{3}$ to 60° and then find the inverse sine of one of their roots in degrees, subtract 60° and divide by 2.

May be implied by a correct value of *x* in degrees or radians.

NB the answers in degrees are: $-143, -6.94, 36.9, 173$ (3sf)

Do **not** allow the mixing of degrees and radians for this mark e.g. $x = \frac{46.1... - \frac{\pi}{3}}{2}$

A1: Any two of awrt $-0.12, -2.5, 0.64$ or $0.65, 3.0$ (Must be in radians)

A1: All four of awrt $-0.121, -2.50, 0.645, 3.02$ and no others in the range.
(Must be in radians)
Condone –2.5 for –2.50 but the others must be awrt as shown for this final mark.

Question Number	Scheme	Marks
10(a)	$x^2+y^2+4x-30y+209=0$ $\Rightarrow (x\pm 2)^2+(y\pm 15)^2 ...=0$	M1
(i)	Centre (–2, 15)	A1
(ii)	Radius $\sqrt{20}$	A1
		(3)
(b)	$y=mx+1 \Rightarrow (x+2)^2+(mx+1-15)^2=20$ or $y=mx+1 \Rightarrow x^2+(mx+1)^2+4x-30(mx+1)+209=0$	M1
	$x^2+m^2x^2+4x-28mx+180=0$ $b^2-4ac=0 \Rightarrow (4-28m)^2-4(1+m^2)\times 180=0$	**d**M1
	$(4-28m)^2-4(1+m^2)180=0 \Rightarrow 16-224m+784m^2-720-720m^2=0$ $\Rightarrow 2m^2-7m-22=0$*	A1*
		(3)
(c)	$2m^2-7m-22=0 \Rightarrow m=\frac{11}{2}, -2$	M1
	$m=\frac{11}{2} \Rightarrow \frac{125}{4}x^2-150x+180=0 \Rightarrow x=\frac{12}{5} \Rightarrow y=\frac{71}{5}$ or $m=-2 \Rightarrow 5x^2+60x+180=0 \Rightarrow x=-6 \Rightarrow y=13$	M1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ **or** $(-6, 13)$ oe	A1
	$\left(\frac{12}{5}, \frac{71}{5}\right)$ **and** $(-6, 13)$ oe	A1
		(4)
		Total 10

(a) Look out for answers to part (a) written in the body of the question.

M1: Attempts to complete the square for **both** variables or states a centre of (±2, ±15)
For completing the square allow $(x\pm 2)^2 ...(y\pm 15)^2 ...=...$

A1: Centre (–2, 15) or e.g. $x=-2$, $y=15$. Condone e.g. –2, 15 or [–2, 15]

A1: For $\sqrt{20}$ or $2\sqrt{5}$ which may be scored following $(x\pm 2)^2+(y\pm 15)^2=20$
Do **not** allow $\pm\sqrt{20}$ but apply isw if a correct radius is seen which is then e.g. converted to a decimal or simplified incorrectly.

(b) Mark (b) and (c) together.

M1: Attempts to substitute $y = mx + 1$ into the given equation or their rearranged equation to obtain an equation in m and x only. Condone slips as long as the intention is clear.

dM1: Attempts $b^2-4ac...0$ where … is "=" or ">" or "<" etc. or equivalent e.g. $b^2=4ac$ where $a=A+Bm^2$, $b=C+Dm$, $c=E$ where A, B, C, D and E are non-zero constants. Condone copying slips if the intention is clear if the above conditions are met. **Depends on the first M mark.**

A1*: Obtains the printed answer with no errors e.g. must have had $b^2-4ac=0$ or equivalent throughout and before the final line e.g. not $b^2-4ac>0$ and brackets expanded. **Note this is a given answer so must follow correct work.**