5. **In this question you must show detailed reasoning.**

(a) Given that x and y are positive numbers such that

$$(x-y)^3 > x^3 - y^3$$

prove that

$$y > x$$

(4)

(b) Using a counter example, show that the result in part (a) is not true for all real numbers.

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 6 marks)

6. (a) Sketch the curve with equation

$$y = a^x + 4$$

where a is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the y-axis
- the equation of the asymptote of the curve

(3)

x	2	2.3	2.6	2.9	3.2	3.5
y	0	0.3246	0.8629	1.6643	2.7896	4.3137

The table shows corresponding values of x and y for

$$y = 2^x - 2x$$

with the values of y given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of y in the given table,

(b) obtain an estimate for $\int_2^{3.5} \left(2^x - 2x\right) \mathrm{d}x$, giving your answer to 2 decimal places.

(3)

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} \left(2^x + 2x\right) \mathrm{d}x$

(ii) $\int_2^{3.5} \left(2^{x+1} - 4x\right) \mathrm{d}x$

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA