Question Number	Scheme	Marks
4.(a)	21	B1 **(1)**
(b)	Sets $f\left(\pm\frac{1}{2}\right)=0 \rightarrow$ equation in k	M1
	Eg. $\left(\frac{1}{2}-2\right)\left(2\times\frac{1}{4}+5\times\frac{1}{2}+k\right)+21=0$ $\Rightarrow 3+k=14 \Rightarrow k=11$ *	A1* **(2)**
(c)	(i) $f(x)=(x-2)(2x^2+5x+11)+21=2x^3+x^2+x-1$	M1
	$=(2x-1)(x^2+x+1)$	dM1, A1
	(ii) Attempts to find the number of roots of their (x^2+x+1)	M1
	• States (x^2+x+1) has no roots with reason	A1
	• Concludes $f(x)=0$ has 1 root (at $x=\frac{1}{2}$)	**(5)** **(8 marks)**

(a)

B1: States 21

(b) If division is attempted in (b), mark (b) and (c) as a whole.

M1: Sets $f\left(\pm\frac{1}{2}\right)=0 \rightarrow$ equation in k. The "= 0" may be implied. If division is used look for setting the remainder after dividing by $(2x - 1)$ equal to zero to form an equation in k.

A1*: Solves a correct equation in k showing at least one correct intermediate line (the "= 0" may be implied but must have been a correct equation preceding the answer). Condone missing brackets if clearly recovered.

May see $k=11$ embedded so $\left(-\frac{3}{2}\right)(3+11)+21=-21+21=0$ ✓ is acceptable.

(c) (i) Only allow work carried out in (b) to count in (c) if carried forward into (c).

M1: Attempts to multiply out $(x-2)(2x^2+5x+11)+21$ to achieve a cubic. If an incorrect value for k is used, allow two M's for equivalent work.

dM1: Attempts to divide by or factor out $(2x-1)$ or $\left(x-\frac{1}{2}\right)$ from their attempt at expanding $(x-2)(2x^2+5x+11)+21$.

Long division – look for the x^2 + extra terms and first subtraction

$$\begin{array}{r} x^2+\ldots \\ 2x-1\overline{)\,"2x^3+x^2+x-1"} \\ "2x^3"-"x^2" \\ \quad "2x^2"+\ldots \\ \quad \ldots. \end{array}$$

Inspection/comparing coefficients – look for correct magnitudes for first and last term in the brackets. $(2x-1)(x^2\pm\ldots\pm1)$

A1: $(2x-1)(x^2+x+1)$ from correct work. Must be brought together in one expression.

Accept $2\left(x-\frac{1}{2}\right)(x^2+x+1)$ but not $\left(x-\frac{1}{2}\right)(2x^2+2x+2)$ Condone if set equal to zero.

Award A0 if they subsequently incorrectly further factorise the quadratic term.

(c)(ii)

M1: Must have attempted to take the correct factor out of their cubic. Attempts to find the number of roots of their (x^2+x+1) **not of** $2x^2+5x+11$ Allow for an attempt **any** of the following:

- an attempt at b^2-4ac
- an attempt at the formula or completing the square
- correct (imaginary) roots from a calculator (must be seen)

Do not accept "no roots" (oe) seen with no evidence for why. Do not accept "Math Error" without an explanation of the error.

A1: Requires all of the following (or equivalents)

- correct factorisation $f(x)=(2x-1)(x^2+x+1)$ or correct separate factors
- correct reason as to why $(x^2+x+1)=0$ hasn't any roots. E.g

  $b^2-4ac=1-4(=-3)<0$ hence no roots. There must be a (correct) calculation, reference to the sign of the discriminant and deduction/implication of no roots, or equivalent via other methods
- concludes $f(x)=0$ has one solution

Ignore erroneous extra statements after a correct reason has been given.

Some examples (following correct factorisation).

Scores M1A1:

$x^2+x+1\rightarrow\Delta=1-4<0$ no real solutions therefore $x=\frac{1}{2}$ is only solution.

$b^2-4ac=-3<0$ so no solutions to x^2+x+1 hence f(x) has only one real solution ($2x$ -1).

We ignore any reference to (2x-1) being the solution.

Scores M1A0:

(ii) one real solution as x^2+x+1 has $b^2-4ac=-3$ so no real solution. *No reference to < 0*

$b^2-4ac=1^2-4(1)(1)=-3\leqslant 0$ so there is only 1 real solution as $x^2+x+1\leqslant 0$ so the only real solution is ($2x$ – 1) which is $x=\frac{1}{2}$ *Needs <0 to be stated*

$\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2}$ no solution, $2x-1$, $x=\frac{1}{2}$ is the only real solution. *Needs either complex roots stated (accepting $\sqrt{-3}$ for 3*i*) or sign of discriminant considered.*

If you see an alternative approach that you think deserves credit, send to review.

Question Number	Scheme	Marks
5 (a)	$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$	M1
	$(x-y)^3 > x^3 - y^3 \Rightarrow -3x^2y + 3xy^2 > 0$	A1
	$\Rightarrow 3xy(y-x) > 0$	dM1
	As x and y are positive numbers $(y-x) > 0 \Rightarrow y > x$ *	A1*
		(4)
(b)	Chooses a suitable counter example. E.g. $x = 3, y = -1$	M1
	Shows that $(3--1)^3 > (3)^3 - (-1)^3$ as $64 > 28$ (BUT $-1 < 3$)	A1
		(2)
		(6 marks)

(a)

M1: Attempts $(x-y)^3 = x^3 \pm Ax^2y \pm Bxy^2 \pm y^3$ but may be unsimplified. Condone at most one slip in an index as long as the middle terms have at least xy in them.

A1: Correct simplified inequality, cubed terms cancelled, other terms may not all be gathered.

E.g $-3x^2y + 3xy^2 > 0$ o.e. Terms need not be all on one side, e.g. $3xy^2 > 3x^2y$

dM1: Takes out a common factor of kxy or divides by kxy. Must be clear division or stated as dividing, but dividing and rearranging to the given answer in one step from $-3x^2y + 3xy^2 > 0$ to $y > x$ is dM0 if method/intermediate step is not stated.

A1*: Requires

- correct working (no incorrect lines) before achieving $y > x$
- reasoning given in the correct/appropriate place,
  - e.g. as x and y are positive so $3xy(y-x) > 0 \Rightarrow (y-x) > 0$

Alt

M1: Attempts $(x-y)^3 = (x-y)(x^2 + axy + y^2)$ **and** $x^3 - y^3 = (x-y)(x^2 + bxy + y^2)$ but may be unsimplified

A1: Both expressions correct $(x-y)^3 = (x-y)(x^2 - 2xy + y^2)$ **and**

$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

dM1: Takes to one side, takes out a common factor $(x-y)$ (dividing by this term is M0) and cancels square terms.

A1*: Requires

- correct working (no incorrect lines) before achieving $y > x$
- reasoning given in the correct/appropriate place,
  - e.g. as x and y are positive so $3xy(y-x) > 0 \Rightarrow (y-x) > 0$

(b)

M1: Chooses a suitable counter example. E.g. $x = 3, y = -1$

Any positive x and negative y will work. Substitution is not needed for this mark. Note that if they choose x and y both positive it is M0.

A1: Shows that the result is not true for their values. As a minimum accept substitution into both sides of the inequality.