7. The height of a river above a fixed point on the riverbed was monitored over a 7-day period.

The height of the river, H metres, t days after monitoring began, was given by

$$H = \frac{\sqrt{t}}{20}(20 + 6t - t^2) + 17 \qquad 0 \leqslant t \leqslant 7$$

Given that H has a stationary value at $t = \alpha$

(a) use calculus to show that α satisfies the equation

$$5\alpha^2 - 18\alpha - 20 = 0$$

(5)

(b) Hence find the value of α, giving your answer to 3 decimal places.

(1)

(c) Use further calculus to prove that H is a maximum at this value of α.

(2)

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Question 7 continued

(Total for Question 7 is 8 marks)

8. (i) A student writes the following statement:

"When a and b are consecutive **prime** numbers, $a^2 + b^2$ is never a multiple of 10"

Prove by counter example that this statement is **not** true.

(2)

(ii) Given that x and y are even integers greater than 0 and less than 6, prove by exhaustion, that

$$1 < x^2 - \frac{xy}{4} < 15$$

(3)

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