Question Number	Scheme	Marks
9(a)	$y = x^3 - 5x^2 + 3x + 14 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 10x + 3 = 0$	M1
	Roots are $3, \dfrac{1}{3} \Rightarrow$ when $x = 3,\ y = "3"^3 - 5\times"3"^2 + 3\times"3" + 14 = ...$	dM1
	Centre is $(3, 5)$	A1
		(3)
(b)	At A $y = 8$	B1
	$r^2 = (2-"3")^2 + ("8"-"5")^2$ (=10)	M1
	$(x-3)^2 + (y-5)^2 = 10$	A1
		(3)
(c)	$\dfrac{"8"-"5"}{2-"3"} = ...(-3)$	M1
	$y - "8" = "\dfrac{1}{3}"(x-2)$	M1
	$y = \dfrac{1}{3}x + \dfrac{22}{3}$ *	A1*
		(3)
(d)	$\int_0^2 x^3 - 5x^2 + 3x + 14\ \mathrm{d}x = ...\left(\dfrac{1}{4}x^4 - \dfrac{5}{3}x^3 + \dfrac{3}{2}x^2 + 14x\right)$	M1
	Area $= \left[\dfrac{1}{4}x^4 - \dfrac{5}{3}x^3 + \dfrac{3}{2}x^2 + 14x\right]_0^2 - \left(\dfrac{1}{2}\times\left(\dfrac{22}{3} + "8"\right)\times 2\right) = ...$ $\dfrac{1}{4}\times 16 - \dfrac{5}{3}\times 8 + \dfrac{3}{2}\times 4 + 14\times 2 - \dfrac{46}{3}$	dM1
	$\dfrac{74}{3} - \dfrac{46}{3} = \dfrac{28}{3}$	A1
		(3)
		(12 marks)

(a)

M1: Differentiates the equation of the curve and sets equal to 0 which may be implied by their attempt to solve below. Do not credit this differentiation in any other parts of the question. For the differentiation, look for at least 2 of $x^3 \to ...x^2,\ -5x^2 \to ...x,\ 3x \to 3$

dM1: Solves a 3 term quadratic equation by any valid means (including a calculator) and substitutes one of their roots in the original equation to find the *y* coordinate. **Depends on the first mark.**

A1: Correct coordinates $(3, 5)$ or e.g. $x = 3$, $y = 5$ (Ignore any work with e.g. $x = \dfrac{1}{3}$ and ignore any work attempting to prove maximum/minimum points – just look for correct coordinates identified)

(b)

B1: *y* coordinate at *A* is 8 (Allow this to score anywhere)

M1: Attempts to find the radius of the circle (or radius^2) using $(2,"8")$ and their minimum point.

Score for $(2-"3")^2 + ("8"-"5")^2$ or equivalent correct work for their coordinates of *T* and *A*.

You can ignore what they call it e.g. condone radius $= (2-"3")^2 + ("8"-"5")^2$

A1: $(x-3)^2 + (y-5)^2 = 10$ oe e.g. $(x-3)^2 + (y-5)^2 = \left(\sqrt{10}\right)^2$, $\sqrt{(x-3)^2 + (y-5)^2} = \sqrt{10}$

(c)

M1: Attempts to find the gradient between *A* and *T* using their coordinates. Note that as the equation of the tangent is given we do not accept −3 just written down.

M1: Attempts to find the equation of the straight line using $x = 2$, their y value at *A* and the negative reciprocal of what they think is the gradient of *AT*.

A1*: $y = \frac{1}{3}x + \frac{22}{3}$ with no errors and both previous method marks scored.

Alternative for first **M**: $(x-3)^2 + (y-5)^2 = 10 \Rightarrow 2(x-3) + 2(y-5)\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3-x}{y-5} = \frac{3-2}{8-5} = \frac{1}{3}$

(d)

M1: Attempts to integrate the curve. Award for a power increasing by 1 on one of the terms.

dM1: A correct method to find the shaded area:

- Substitutes in 2 (and 0) into the integrated function and subtracts either way round. May not see 0 substituted in.
- Attempts to find the shaded area by subtracting the area of the trapezium $\frac{1}{2}\times\left(\frac{22}{3} + \text{"8"}\right)\times 2$ from their area under the curve. It is dependent on the previous method mark.

It must be a correct method for the area of the trapezium e.g. $\frac{1}{2}\times\left(\frac{22}{3} + \text{their } y \text{ at } A\right)\times 2$

Or e.g. rectangle + triangle: $2\times\frac{22}{3} + \frac{1}{2}\times 2\left(\text{their } y \text{ at } A - \frac{22}{3}\right)$

Or by integration $\int_0^2 \left(\frac{1}{3}x + \frac{22}{3}\right)\mathrm{d}x = \left[\frac{1}{6}x^2 + \frac{22}{3}x\right]_0^2 = \frac{2}{3} + \frac{44}{3} = \frac{46}{3}$

A1: $\frac{28}{3}$ or exact equivalent cso

Alt(d)

M1: Attempts to integrate (curve – line) or (line – curve).
Award for the power increasing by 1 on one of the terms.

dM1: A correct method to find the shaded area.
Substitutes 2 (and 0) into the integrated function and subtracts either way round:

$$\int_0^2 x^3 - 5x^2 + \frac{8}{3}x + \frac{20}{3}\,\mathrm{d}x = \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{4x^2}{3} + \frac{20x}{3}\right]_0^2 = \frac{2^4}{4} - \frac{5\times 2^3}{3} + \frac{4\times 2^2}{3} + \frac{20\times 2}{3} = \ldots$$

Or e.g. $\int_0^2 x^3 - 5x^2 + 3x + 14 - \frac{1}{3}x - \frac{22}{3}\,\mathrm{d}x = \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 14x - \frac{x^2}{6} - \frac{22x}{3}\right]_0^2 =$

$$= \frac{2^4}{4} - \frac{5\times 2^3}{3} + \frac{3\times 2^2}{2} + 14\times 2 - \frac{2^2}{6} - \frac{22\times 2}{3} = \ldots$$

You can condone slips when collecting terms or for a slip with brackets e.g. attempting:

$$\int_0^2 x^3 - 5x^2 + 3x + 14 - \frac{1}{3}x + \frac{22}{3}\,\mathrm{d}x$$

A1: $\frac{28}{3}$ or exact equivalent cso

With otherwise correct work leading to $-\frac{28}{3}$ allow full marks if then made positive.

Question Number	Scheme	Marks
10(i)(a)	$2a$	B1
(b)	$\log_2\left(\frac{\sqrt{3}}{16}\right)=\log_2\sqrt{3}-\log_2 16,\ =\frac{1}{2}a-4$	M1A1
		(3)

(i)(a)
B1: $2a$
(b)

M1: Uses the subtraction rule for logs to write $\log_2\left(\frac{\sqrt{3}}{16}\right)=\log_2\sqrt{3}-\log_2 16$ seen or implied.

A1: $\frac{1}{2}a-4$

Do not isw here so if they reach $\frac{1}{2}a-4$ correctly and then say $=a-8$ score M1A0

(ii) This general guidance will apply to most cases you will come across:

M1: Takes logs (same base) of **both** sides and applies the addition rule for logs on the lhs
See possible rearrangements below which may be seen before the addition rule is applied.

dM1: Correct method to make x the subject. Condone slips in rearrangement but there must be more than one term in x. Award for collecting terms in x on one side and non x terms on the other side, factorising and then dividing. **Dependent on the first method mark.**

B1: The correct power law seen for logs. Allow this mark to score independently so sight of e.g. $\log_2 2^x=x\log_2 2,\ \log_2 3^x=x\log_2 3,\ \log_2 6^x=x\log_2 6,\ \log_2 2^{x+4}=(x+4)\log_2 2$ etc. can score this mark. (Condone e.g. $\log_2 2^{x+4}=x+4\log_2 2$ for this mark)

Note that this may follow incorrect work e.g.

$$3^x\times 2^{x+4}=6\Rightarrow \log_2 3^x\times\log_2 2^{x+4}=\log_2 6\Rightarrow x\log_2 3\times(x+4)\log_2 2^{x+4}=\log_2 6$$

A1: $\frac{a-3}{a+1}$ or e.g. $\frac{3-a}{-a-1}$

Note that candidates are unlikely to be working in anything other than base 2 so you can condone the omission of the base throughout but see 3rd below the main scheme below.

Note that for reference, $\frac{a-3}{a+1}=-0.54741122...$ which may be useful in some circumstances.

(ii)	Examples: $3^x \times 2^{x+4} = 6 \Rightarrow \log_2 3^x + \log_2 2^{x+4} = \log_2 6$ or $3^x \times 2^{x+4} = 6 \Rightarrow 3^x \times 2^x \times 2^4 = 6 \Rightarrow \log_2 3^x + \log_2 2^x + \log_2 2^4 = \log_2 6$ or $3^x \times 2^{x+4} = 6 \Rightarrow 3^x \times 2^{x+3} = 3 \Rightarrow \log_2 3^x + \log_2 2^{x+3} = \log_2 3$ or $3^x \times 2^{x+4} = 6 \Rightarrow 3^x \times 2^x = \frac{3}{8} \Rightarrow \log_2 3^x + \log_2 2^x = \log_2 \frac{3}{8}$ or $3^x \times 2^{x+4} = 6 \Rightarrow 3^{x-1} \times 2^{x+3} = 1 \Rightarrow \log_2 3^{x-1} + \log_2 2^{x+3} = \log_2 1$	M1
	Examples: $x\log_2 3 + (x+4)\log_2 2 = \log_2 6 \Rightarrow x(\log_2 3 + \log_2 2) = \log_2 6 - 4 \Rightarrow x = ...$ or $x\log_2 3 + x\log_2 2 + 4 = \log_2 6 \Rightarrow x(\log_2 3 + \log_2 2) = \log_2 6 - 4 \Rightarrow x = ...$ or $x\log_2 3 + (x+3)\log_2 2 = \log_2 3 \Rightarrow x(\log_2 3 + \log_2 2) = \log_2 3 - 3\log_2 2 \Rightarrow x = ...$ or $x\log_2 3 + x\log_2 2 = \log_2 \frac{3}{8} \Rightarrow x(\log_2 3 + \log_2 2) = \log_2 \frac{3}{8} \Rightarrow x = ...$ or $(x-1)\log_2 3 + (x+3)\log_2 2 = 0 \Rightarrow ax + x = a - 3 \Rightarrow x = ...$	dM1
	$\log_2 a^b = b\log_2 a$	B1
	$x = \frac{a-3}{a+1}$	A1
		(4)
		(7 marks)

Some alternatives you may come across are below:

Alternative not requiring the addition law:

$$3^x \times 2^{x+4} = 6 \Rightarrow 3^x \times 2^x = \frac{3}{8} \Rightarrow \log_2 3^x \times 2^x = \log_2 \frac{3}{8} \Rightarrow \log 6^x = \log_2 \frac{3}{8}$$

$$\log 6^x = \log_2 \frac{3}{8} \Rightarrow x\log_2 6 = \log_2 \frac{3}{8} \Rightarrow x = ...$$

$$x = \frac{\log_2 \frac{3}{8}}{\log_2 6} = \frac{\log_2 3 - \log_2 8}{\log_2 6} = \frac{a-3}{a+1}$$

Score as:

M1: Divides by 2^4, takes logs of **both** sides and writes $3^x \times 2^x$ as 6^x

B1: $\log_2 6^x = x\log_2 6$

dM1: Makes x the subject

A1: $\frac{a-3}{a+1}$ or e.g. $\frac{3-a}{-a-1}$