Question	Scheme	Marks
8(i)	$3\sin(\theta+30°)=7\cos(\theta+30°)\Rightarrow\tan(\theta+30°)=\frac{7}{3}$	B1
	$\theta+30°=\arctan\left(\frac{7}{3}\right)\ (=66.8°)$	M1
	$\theta=36.8°$ or $216.8°$ (awrt)	A1A1
		(4)
(ii)	(a) $3\sin^3 x=5\sin x-7\sin x\cos x\Rightarrow 3\sin x\left(1-\cos^2 x\right)=5\sin x-7\sin x\cos x$	M1
	$\Rightarrow\sin x\left(5-7\cos x+3\cos^2 x-3\right)=0$	
	$\Rightarrow\sin x\left(3\cos^2 x-7\cos x+2\right)=0$	A1
	(b) $\sin x=0\Rightarrow x=0$	B1
	$\Rightarrow\sin x\left(3\cos x-1\right)\left(\cos x-2\right)=0\Rightarrow\cos x=...$	M1
	$3\cos x=1\Rightarrow x=\arccos\left(\frac{1}{3}\right)=(\pm)1.23$	M1
	Both of x = awrt -1.23, 1.23	A1
		(6)
		(10 marks)

Notes:

(i)

Note: answers only scores no marks.

B1: Divides through by $\cos(\theta+30°)$ to get correct equation in tan.

M1: Applies arctan to find a value for $\theta+30°$. If using other approaches it is for correct work to achieve one value for $\theta+30°$ or just θ as appropriate. (Allow if "radians" are used.)

A1: One of awrt $36.8°$ or awrt $216.8°$

A1: Both of awrt $36.8°$ and awrt $216.8°$ with no other solutions in the range.

(ii)(a)

Part (ii) may be marked as a whole.

M1: Uses $\sin^2 x=1-\cos^2 x$ in the equation and gathers terms on one side and factors out the sin x (Allow the M if the sin x is cancelled) May be seen in (b)

A1: Achieves suitable form. Accept non-zero multiples of the quadratic in cos x. (Allow if sin x is cancelled and put back in later). Result must be clearly stated but may be seen in (b).

(b)

B1: For $x = 0$ as one solution.

M1: Solves the quadratic in cos x, usual rules (may be implied by correct answers).

M1: Takes arccos of at least one value that has modulus less than 1 which is a root of their quadratic. Allow if in degrees, but must reach a value.

A1: Both solutions from cosine equation correct and no others in the range.

	Alternatives to (i)	
8(i) Alt 1	$\sin(\theta+30°)=\frac{7}{\sqrt{58}}$ or $\cos(\theta+30°)=\frac{3}{\sqrt{58}}$	**B1**
	$(3\sin(\theta+30°))^2=7(\cos(\theta+30°))^2 \Rightarrow ..\sin^2(\theta+30°)=..(1-\sin^2(\theta+30°))$ $\Rightarrow \sin(\theta+30°)=..\Rightarrow \theta+30°=...$ Or $(3\sin(\theta+30°))^2=7(\cos(\theta+30°))^2 \Rightarrow ..(1-\cos^2(\theta+30°)=..\cos^2(\theta+30°)$ $\Rightarrow \cos(\theta+30°)=..\Rightarrow \theta+30°=...$	**M1**
	$\theta=36.8°$ or $216.8°$ (awrt)	**A1A1**
		(4)

Alt:

B1: For reaching $\sin(\theta+30°)=\frac{7}{\sqrt{58}}$ or $\cos(\theta+30°)=\frac{3}{\sqrt{58}}$

M1: For a complete method of squaring both sides, applying $\sin^2(...)+\cos^2(...)=1$ to achieve an equation in just sin or cos and then applying the inverse function to their solution to achieve one value for $\theta+30°$ or just θ as appropriate. . (Allow if "radians" are used.)

A1A1: As above, must have rejected any extra solutions from squaring.

8(i) Alt 2	$3\sin(\theta+30°)=7\cos(\theta+30°)=\frac{3\sqrt{3}}{2}\sin\theta+\frac{3}{2}\cos\theta=\frac{7\sqrt{3}}{2}\cos\theta-\frac{7}{2}\sin\theta$ oe	**B1**
	$3\sin(\theta+30°)=7\cos(\theta+30°)\Rightarrow$ $3(\sin\theta\cos30°\pm\cos\theta\sin30°)=7(\cos\theta\sin30°\pm\sin\theta\sin30°)$ $\Rightarrow \frac{\sin\theta}{\cos\theta}=\tan\theta=...\Rightarrow\theta=\arctan(..)=...$	**M1**
	$\theta=36.8°$ or $216.8°$ (awrt)	**A1A1**
		(4)

Alt 2:

B1: Correct application of compound angle formula and trig ratios to reach

$\frac{3\sqrt{3}}{2}\sin\theta+\frac{3}{2}\cos\theta=\frac{7\sqrt{3}}{2}\cos\theta-\frac{7}{2}\sin\theta$ **oe**

M1: Attempts compound angle formula (allow sign slips), rearranges to make $\tan\theta$ the subject and applies arctan to find at least one value for θ . (Allow if "radians" are used.)

A1A1: As main scheme.

Question	Scheme	Marks
9(a)	$V = hl^2 \Rightarrow 250000 = hl^2$ or $l = \dfrac{500}{\sqrt{h}}$ oe (may use e.g. $hl = \sqrt{250\,000h}$)	B1
	$S = l^2 + 4hl = \text{"}\dfrac{250000}{h}\text{"} + 4h \times \text{"}\dfrac{500}{\sqrt{h}}\text{"}$	M1
	$S = \dfrac{250000}{h} + 2000\sqrt{h}$ *	A1*
		(3)
(b)	$\dfrac{\mathrm{d}S}{\mathrm{d}h} = -\dfrac{250000}{h^2} + 2000 \times \dfrac{1}{2}h^{-\frac{1}{2}}$ oe	M1A1
	$\dfrac{\mathrm{d}S}{\mathrm{d}h} = 0 \Rightarrow -\dfrac{250000}{h^2} + 2000 \times \dfrac{1}{2}h^{-\frac{1}{2}} = 0 \Rightarrow h^k = ...$	dM1
	$\Rightarrow h^{\frac{3}{2}} = 250 \Rightarrow h = ...$	ddM1
	$h = 250^{\frac{2}{3}}$	A1
		(5)
(c)	$\dfrac{\mathrm{d}^2S}{\mathrm{d}h^2} = \dfrac{500000}{h^3} - 500h^{-\frac{3}{2}}$	M1
	$\left.\dfrac{\mathrm{d}^2S}{\mathrm{d}h^2}\right\|_{h=39.7} = 6 > 0$ hence gives the minimum value.	A1
		(2)
		(10 marks)

Notes:

(a)

B1: Correct equation linking h and l seen or implied by substitution.

M1: Attempts the surface area and substitutes to eliminate l. Allow $l^2 + 2(hl + hl)$ or $2(l^2 + hl + hl)$ (or equivalents) for the attempt at surface area.

A1*: Achieves the given result with no errors.

(b)

Allow if $\dfrac{\mathrm{d}S}{\mathrm{d}h}$ is missing or called $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ throughout.

M1: Attempts derivative of the area function – power decreased by one in at least one term.

A1: Correct derivative. Need not be simplified.

dM1: Sets derivative equal to zero and attempts to solve as far as a power of h equal to something. Depends on first M

ddM1: Depends on both M marks, attempts to solve an equation with a fractional index to find h

A1: $h = 250^{\frac{2}{3}}$ or states $k = \dfrac{2}{3}$