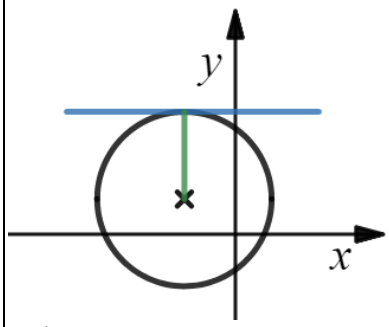
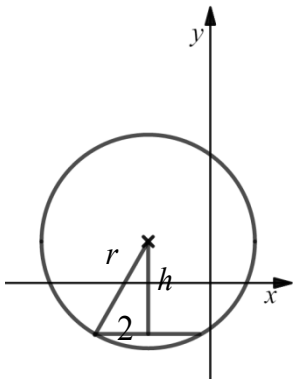
| Question Number | Scheme | Marks |
|---|---|---|
| **6.(a)** | $x^2+y^2+6x-4y-14=0$ | |
| | Attempts $(x\pm3)^2+(y\pm2)^2$ .....$(=0)$ | M1 |
| | (i) Centre $(-3,2)$ | A1 |
| | (ii) Radius $\sqrt{27}$ or $3\sqrt{3}$ | A1 **(3)** |
| **(b)** |   Attempts $y/k="2"\pm"3\sqrt{3}"$ | M1 |
| | Both $k=2+3\sqrt{3}$ and $k=2-3\sqrt{3}$ | A1 **(2)** |
| | Alt: | |
| | $y=k\Rightarrow x^2+6x+k^2-4k-14=0$ | |
| | $\Rightarrow b^2-4ac=0\Rightarrow 6^2-4\times1\times\left(k^2-4k-14\right)=0$ | |
| | $\Rightarrow 4k^2-16k-92=0\Rightarrow k=...$ | M1 |
| | $k=2\pm3\sqrt{3}$ $(oe)$ | A1 **(2)** |
| **(c)** |   Attempts Pythagoras to find $h$ $h^2=r^2-2^2$ | M1 |
| | Attempts $p=2-h$ $p=2-\sqrt{23}$ (oe) | dM1 A1 **(3)** |
| | Alt1: chord touches circle when $x=-1$ (or $-5$) or need $(x+3)=\pm2$ | M1 |
| | $\Rightarrow 2^2+(y-2)^2=27\Rightarrow y=...$ or $(-1)^2+y^2+6(-1)-4y-14=0\Rightarrow y=...$ | dM1 |
| | $p=2-\sqrt{23}$ | A1 **(3)** |
| | Alt2: Sets $y=p$ and applies difference of roots to be 4 | |
| | $\Rightarrow x^2+6x+p^2-4p-14=0\Rightarrow x=\dfrac{-6\pm\sqrt{36-4(p^2-4p-14)}}{2}$ | M1 |
| | $\Rightarrow 2\sqrt{23+4p-p^2}=4$ | dM1 |
| | $\Rightarrow p^2-4p-19=0\Rightarrow p=...$ | A1 |
| | $p=2-\sqrt{23}$ | **(3)** **(8 marks)** |

(a)

M1 Attempts to complete the square. Look for $(x \pm 3)^2 + (y \pm 2)^2 ..... = 0$

M1 may be awarded for a centre of $(\pm 3, \pm 2)$ (from no, or from incorrect, working)

A1 Centre $(-3, 2)$

A1 Radius $\sqrt{27}$ or $3\sqrt{3}$

(b)

M1 Attempts one value of $k$ by $"2" \pm "r"$ . In the alternative it is for setting $y = k$ in the circle equation, applying $b^2 - 4ac = 0$ and then solving the resulting quadratic in $k$, or equivalent method. But substitution of $x$=0 is M0.

A1 For both $k = 2 + 3\sqrt{3}$ and $k = 2 - 3\sqrt{3}$ Allow $y \leftrightarrow k$ Accept exact equivalents, e.g. with $\sqrt{27}$

Alt by differentiation requires

M1 Achieves $ax + by\frac{dy}{dx} + cx + d\frac{dy}{dx} = 0$ followed by setting $\frac{dy}{dx} = 0$ , finding $x$ and substituting into the circle equation then attempting to solve the quadratic. A1 Correct answers.

(c)

M1 For an attempt to use Pythagoras's theorem with 2 and their '$r$' to find '$h$' or 'h'$^2$

dM1 Attempts $p = "2" \pm "h"$ Condone $p \leftrightarrow y$ Follow through their $y$ coordinate of the centre.
Can be implied by the correct decimal answer (awrt −2.8)

A1 $p = 2 - \sqrt{23}$ only Condone $p \leftrightarrow y$ Accept equivalent exact forms e.g $-\left(\sqrt{23} - 2\right)$

Alternatives are possible:

Alt 1

M1 Identifies an $x$ coordinate where the chord touches the circle, $x = -3 \pm 2$, or applies $x + 3 = \pm 2$

dM1 Substitutes into the circle equation (either form) and attempts to solve the quadratic (usual rules)

A1 $p = 2 - \sqrt{23}$ only Condone $p \leftrightarrow y$

Alt2

M1 Substitutes $y = p$ into $C$ and applies the difference between roots is 4

dM1 Solves the resulting quadratic to find $p$.

A1 $p = 2 - \sqrt{23}$ only Condone $p \leftrightarrow y$

| Question Number | Scheme | Marks |
|---|---|---|
| **7 (a)** | $8\tan\theta = 3\cos\theta$ | |
| | Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta} \rightarrow$ $\quad 8\dfrac{\sin\theta}{\cos\theta} = 3\cos\theta$ | M1 |
| | $8\sin\theta = 3\cos^2\theta$ oe | |
| | Uses $\cos^2\theta = 1-\sin^2\theta \rightarrow$ $\quad 8\sin\theta = 3\left(1-\sin^2\theta\right)$ | M1 |
| | $3\sin^2\theta + 8\sin\theta - 3 = 0$ * | A1 * |
| | | **(3)** |
| **(b)** | $(3\sin 2x - 1)(\sin 2x + 3) = 0$ | M1 |
| | Critical value(s) of $\dfrac{1}{3}, (-3)$ | A1 |
| | Correct method to find $x$ from their $\sin 2x = \dfrac{1}{3}$ | dM1 |
| | $x =$ awrt $9.74°, 80.26°$ | A1 |
| | | **(4)** |
| | | **(7 marks)** |

(a)

M1 Uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (oe) in $8\tan\theta = 3\cos\theta$ . Condone slips in coefficients.

M1 Multiplies by $\cos\theta$ and uses $\cos^2\theta = 1-\sin^2\theta$ (oe) to produce an equation in just $\sin\theta$

A1* Proceeds to $3\sin^2\theta + 8\sin\theta - 3 = 0$ with no arithmetical or notational errors.
No mixed variables within the lines of the "proof"
An example of a notational error is $\cos\theta^2$ for $\cos^2\theta$ (Note that this would not lose the M1)

(b)

M1 Attempts to find the critical values of the given quadratic. Implied by correct values given if no working shown. Allow if just $\dfrac{1}{3}$ is given.

A1 Critical value of $\dfrac{1}{3}$ and no incorrect second value. The $-3$ need not be stated. Accept awrt 0.333 for $\dfrac{1}{3}$

dM1 A correct method to find one value of $x$ from their $\sin 2x = \dfrac{1}{3}$

It requires both arcsin and $\div 2$ and may be implied by one answer to one decimal place rounded or truncated eg awrt 9.7 or awrt 80.3 (or for 2sf radian answer awrt 0.17, or awrt 1.4)

A1 $x =$ awrt $9.74°, 80.26°$ . A0 if extra solutions are given in the range but ignore solutions outside.

Note that M1A0M1A1 is possible if correct answers come from a critical value of $\dfrac{1}{3}$ if a second incorrect C.V. was also found but does not lead to extra solutions in the range.