Question Number	Scheme	Marks
5.(a)	$\frac{dP}{dx} = 12 - \frac{3}{2}x^{\frac{1}{2}}$	M1A1
	Sets $\frac{dP}{dx} = 0 \rightarrow 12 - \frac{3}{2}x^{\frac{1}{2}} = 0 \rightarrow x^n = \ldots$	dM1
	$x = 64$	A1
	When $x = 64 \Rightarrow P = 12 \times 64 - 64^{\frac{3}{2}} - 120 = \ldots$	M1
	Profit = (£) 136 000	A1
		(6)
(b)	$\left(\frac{d^2P}{dx^2}\right) = -\frac{3}{4}x^{-\frac{1}{2}}$ and substitutes in their $x = 64$ to find its value or state its sign	M1
	At $x = 64$ $\frac{d^2P}{dx^2} = -0.09375 < 0 \Rightarrow$ maximum	A1
		(2)
		(8 marks)

Notes

You should mark parts a and b together. You may see work in (a) from (b)

(a)

M1 Attempts to differentiate $x^n \rightarrow x^{n-1}$ seen at least once. It must be an x term and **not** the $120 \rightarrow 0$

A1 $\frac{dP}{dx} = 12 - \frac{3}{2}x^{\frac{1}{2}}$ with no need to see the lhs. Condone $\frac{dy}{dx}$ all of the way through part (a).

dM1 Sets their $\frac{dP}{dx} = 0$ and proceeds to $x^n = k, k > 0$. Dependent upon the previous M. Don't be too concerned with the mechanics of process. Condone an attempted solution of $\frac{dP}{dx} \ldots 0$ where **...** could be an inequality

A1 $x = 64$. Condone $x = \pm 64$ here

M1 Substitutes their solution for $\frac{dP}{dx} = 0$ into P and attempts to find the value of P.

The value of x must be positive. If two values of x are found, allow this mark for any attempt using a positive value.

A1 CSO. Profit = (£) 136 000 or 136 thousand but not 136 or P = 136.

This cannot follow two values for x, eg $x = \pm 64$ Condone a lack of units or incorrect units such as $

(b)

M1 Achieves $\frac{d^2P}{dx^2} = kx^{-\frac{1}{2}}$ and attempts to find its value at $x = ''64''$

Alternatively achieves $\frac{d^2P}{dx^2} = kx^{-\frac{1}{2}}$ and attempts to state its sign. Eg $\frac{d^2P}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}} < 0$

Allow $\frac{d^2P}{dx^2}$ appearing as $\frac{d^2y}{dx^2}$ for the both marks.

A1 Achieves $x = 64$, $\frac{d^2P}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$ and states $\frac{d^2P}{dx^2} = -\frac{3}{32} < 0$ (at $x = 64$) then the profit is maximised.

This requires the correct value of x, the correct value of the second derivative (allowing for awrt -0.09) a reason + conclusion.

Alt: Achieves $x = 64$, $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = -\frac{3}{4}x^{-\frac{1}{2}}$ and states as $x > 0$ or $\sqrt{x} > 0$ means that $\frac{\mathrm{d}^2P}{\mathrm{d}x^2} < 0$ then the profit is maximised.

Part (b) merely requires the use of calculus so allow

M1 Attempting to find the value of $\frac{\mathrm{d}P}{\mathrm{d}x}$ at two values either side, but close to their 64. Eg. For 64 , allow the lower value to be $63.5 \leqslant x < 64$ and the upper value to be $64 < x \leqslant 64.5$

A1 Requires correct values, correct calculations with reason and conclusion

Question Number	Scheme	Marks
6.(a)	Sets $f(3)=0 \rightarrow$ equation in k Eg. $27k-135-96-12=0$ $\Rightarrow 27k=243 \Rightarrow k=9$ * (= 0 must be seen)	M1 A1* **(2)**
(b)	$9x^3-15x^2-32x-12=(x-3)\left(9x^2+12x+4\right)$ $=(x-3)(3x+2)^2$	M1 A1 dM1 A1 **(4)**
(c)	Attempts $\cos\theta=-\frac{2}{3}$ $\theta=131.8°, 228.2°$ (awrt)	M1 A1 **(2)** **(8 marks)**

Notes

(a)

M1 Attempts to set $f(3)=0 \rightarrow$ equation in k Eg. $27k-135-96-12=0$. Condone slips.

Score when you see embedded values within the equation or two correct terms on the lhs of the equation. It is implied by sight of $27k-243=0$ or $27k=135+96+12$.

A1* Completes proof with at least one intermediate "solvable" line namely $27k=243 \Rightarrow k=9$ or $27k-243=0 \Rightarrow k=9$. This is a given answer so there should be no errors.

It is a "show that" question so expect to see

(i) Either $f(3)=0$ explicitly stated or implied by sight of $27k-135-96-12=0$ or $27k-243=0$

(ii) One solvable intermediate line followed by $k=9$

.....

A candidate could use $k=9$ and start with $f(x)=9x^3-15x^2-32x-12$

M1 For attempting $f(3)=9\times3^3-15\times3^2-32\times3-12$.

Alt attempts to divide $f(x)$ by (*x*-3). See below on how to score such an attempt

A1* Shows that $f(3)=0$ and makes a minimal statement to the effect that "so $k=9$"

If division is attempted it must be correct and a statement is required to the effect that there is no remainder, " so $k=9$"

.....

If candidates have divided (correctly) in part (a) they can be awarded the first two marks in (b) when they start factorsing the $9x^2+12x+4$ term.

(b)

M1 Attempt to divide or factorise out $(x-3)$. Condone students who use a different value of *k*.

For factorisation look for first and last terms $9x^3-15x^2-32x-12=(x-3)\left(\pm9x^2.....\pm4\right)$

For division look for the following line $x-3\overline{)9x^3-15x^2-32x-12}$ with $9x^2+.....$ and $\underline{9x^3-27x^2}$

A1 Correct quadratic factor $9x^2+12x+4$.

You may condone division attempts that don't quite work as long as the correct factor is seen.