(b)	$\frac{1}{3}(2x+3)^2(x-4)=2x-12 \Rightarrow (4x^2+12x+9)(x-4)=6x-36$ $\Rightarrow 4x^3-16x^2+12x^2-48x+9x-36=6x-36$ $\Rightarrow 4x^3-4x^2=45x$ or e.g. $4x^3-4x^2-45x=0$	M1A1
	$x(4x^2-4x-45)=0 \Rightarrow x=\frac{4\pm\sqrt{736}}{8}$	**d**M1
	$=\frac{1\pm\sqrt{46}}{2}$	A1
		(4)
		(7 marks)

(b) Notes

(b)

M1: Sets their $f(x)$ **which must be a cubic function** equal to $2x-12$ and starts to simplify by expanding the lhs and collecting terms on lhs or from rhs to lhs.
Note that the lhs may already be expanded e.g. if the alternative was used in part (a).

A1: Correct simplified 3 term equation $4x^3-4x^2-45x=0$ or $4x^2-4x-45=0$
oe e.g. $\frac{4}{3}x^3-\frac{4}{3}x^2-15x=0$ or $\frac{4}{3}x^2-\frac{4}{3}x-15=0$
The terms do not necessarily need to be all on one side e.g. $\frac{4}{3}x^2-\frac{4}{3}x=15$ is acceptable.
The "= 0" may be implied by their attempt to solve.

dM1: Solves the resulting 3 term equation which must either be a 3TQ or a 3 term cubic equation with no constant term.
The solution of their equation can be by any method including a calculator.
If values are just written down you may need to check.
Note that it is acceptable to go from e.g. $4x^3-4x^2-45x=0$ to the correct roots using a calculator.
Note that a cubic equation of the required form may have been obtained fortuitously but this mark is still available e.g. if they obtained $(f(x)=)(2x+3)^2(x-4)+24$ in part (a).

A1: Correct values $\frac{1\pm\sqrt{46}}{2}$ oe e.g. $\frac{1}{2}\pm\frac{\sqrt{46}}{2}$
Condone the inclusion of $x = 0$
Ignore any attempts to find the y values.

Question Number	Scheme	Marks
7(a)	$\frac{1}{2}r^2\theta = 1$ **or** $2r + r\theta = 8$	B1
	$\frac{1}{2}r^2\theta = 1$ **and** $2r + r\theta = 8$	B1
		(2)

Notes

Mark (a) and (b) together i.e. ignore any labelling.

Note that e.g. $\frac{\theta}{360} \times \pi r^2 = 1$ **is incorrect for the area equation.**

B1: Either $\frac{1}{2}r^2\theta = 1$ **or** $2r + r\theta = 8$ or their exact equivalents e.g. $\frac{1}{2} \times r^2\theta = 1$ or $2 \times r + r \times \theta = 8$

Ignore any units included in the equations e.g. $\frac{1}{2}r^2\theta = 1\text{cm}^2$ or e.g. $2r + r\theta = 8\text{cm}$

B1: Both $\frac{1}{2}r^2\theta = 1$ **and** $2r + r\theta = 8$ or their exact equivalents e.g. $\frac{1}{2} \times r^2\theta = 1$ or $2 \times r + r \times \theta = 8$

Ignore any units included in the equations e.g. $\frac{1}{2}r^2\theta = 1\text{cm}^2$ or e.g. $2r + r\theta = 8\text{cm}$

Note that these equations may be seen in the body of the question and (less likely) may be implied if they answer (b) without writing them down explicitly e.g. $2r + r\left(\frac{2}{r^2}\right) = 8$ or $\frac{1}{2}\left(\frac{8}{2+\theta}\right)^2\theta = 1$

(b)	$\frac{1}{2}r^2\theta = 1 \Rightarrow \theta = \frac{2}{r^2}$, $2r + \frac{2}{r} = 8$ or $2r + r\theta = 8 \Rightarrow r = \frac{8}{2+\theta}$, $\frac{1}{2}r^2\theta = 1 \Rightarrow \frac{1}{2}\left(\frac{8}{2+\theta}\right)^2\theta = 1$	M1
	$r^2 - 4r + 1 = 0$ oe or $\theta^2 - 28\theta + 4 = 0$ oe	A1
	e.g. $(r-2)^2 = 3 \Rightarrow r =$ or e.g. $\theta = \frac{28 \pm \sqrt{28^2 - 4\times1\times4}}{2}$	**d**M1
	$\theta = \frac{2}{r^2} = \frac{2}{\left(2+\sqrt{3}\right)^2} = \frac{2}{7+4\sqrt{3}} = \frac{2}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = ...\left(14-8\sqrt{3}\right)$ or e.g. $r = \frac{8}{2+\theta} = \frac{8}{2+14-8\sqrt{3}} = \frac{8\left(16+8\sqrt{3}\right)}{\left(16-8\sqrt{3}\right)\left(16+8\sqrt{3}\right)} = ...\left(2+\sqrt{3}\right)$	**dd**M1
	$r = 2+\sqrt{3}$, $\theta = 14-8\sqrt{3}$	A1
		(5)

M1: From 2 equations in r and θ, attempts to create an equation in a single variable.

The equations must be dimensionally correct but note that **this** mark is available even if the resulting equation in one variable is linear e.g. from $\frac{1}{2}r^2\theta = 1$ and $r\theta = 8$

You do not need to be concerned about the mechanics as long as they reach an equation in one variable.

A1: Any correct 3 term quadratic equation in a single variable with terms collected but not necessarily all on one side e.g. $2r^2-8r+2=0$ o.e e.g. $r^2-4r=-1$ or $\theta^2-28\theta+4=0$ oe e.g. $\theta^2-28\theta=-4$ The "= 0" may be implied by their attempt to solve.
May be seen embedded e.g. $r\left(2r^2-8r+2\right)=0$

dM1: Solves their 3TQ in r or θ using a **non-calculator** method to produce a surd solution.
See General Guidance for solving a 3TQ.
Note that just quoting the quadratic formula followed by values scores M0 but that e.g. $\theta=\frac{28\pm\sqrt{28^2-4\times1\times4}}{2}=14\pm8\sqrt{3}$ or e.g. $r=\frac{4\pm\sqrt{4^2-4\times1\times1}}{2}=2\pm\sqrt{3}$ is sufficient.

ddM1: Dependent upon the previous M, it is for a correct method to find the other variable from a fraction of the form $\frac{\ldots}{a+b\sqrt{c}}$ where a, b and c are non-zero and $\sqrt{c}$ is irrational using a **non-calculator** method. E.g. multiplies numerator and denominator by the conjugate of the denominator. Note that having found θ, any progress to find r using $r=\sqrt{\frac{2}{\theta}}$ is unlikely as they will have to use a non-calculator method. This mark is for finding r or θ using a correct method and not for finding a value for e.g. r^2.

A1: $r=2+\sqrt{3},\ \theta=14-8\sqrt{3}$ o.e. such as $r=2+\sqrt{3},\ \theta=14-2\sqrt{48}$
Condone the omission of "$r=$" and/or "$\theta=$" if it is clear which is which.
Note that once the rationalisation of the denominator process is shown e.g. $\frac{2}{7+4\sqrt{3}}\times\frac{7-4\sqrt{3}}{7-4\sqrt{3}}$ or $\frac{8\left(16+8\sqrt{3}\right)}{\left(16-8\sqrt{3}\right)\left(16+8\sqrt{3}\right)}$, it is acceptable to write down the final answers.
If there are any extra incorrect values then withhold this mark.

		(7 marks)

Note that in 7(b), some candidates solve simultaneously using $r=\sqrt{\frac{2}{\theta}}$ e.g.

$$\frac{1}{2}r^2\theta=1\Rightarrow r=\sqrt{\frac{2}{\theta}}\Rightarrow\theta\sqrt{\frac{2}{\theta}}+2\sqrt{\frac{2}{\theta}}=8$$

This scores M1 for an equation in one variable and then A1 for

$$\theta\sqrt{\frac{2}{\theta}}+2\sqrt{\frac{2}{\theta}}=8\Rightarrow\sqrt{2}\theta-8\sqrt{\theta}+2\sqrt{2}=0 \text{ oe}$$

i.e. a correct quadratic equation in $\sqrt{\theta}$

They will need to solve this for $\sqrt{\theta}$ using a non-calculator method e.g.

$$\sqrt{2}\theta-8\sqrt{\theta}+2\sqrt{2}=0\Rightarrow\sqrt{\theta}=\frac{8\pm\sqrt{64-16}}{2\sqrt{2}}=\frac{8\pm4\sqrt{3}}{2\sqrt{2}}$$

and then square to obtain a value for θ for **d**M1

$$\theta=\left(\frac{8\pm4\sqrt{3}}{2\sqrt{2}}\right)^2=14\pm8\sqrt{3}$$

and then **dd**M1A1 as in the main scheme.