(c)	$y \leqslant -2x^2 + 12x + 7$, $y \geqslant "\frac{25}{3}"x$, $x \geqslant 0$	B1ft B1ft
		(2)
		(7 marks)

(c) Notes

Allow use of f(x) for y.

Do not allow use of R for y e.g. $R \leqslant -2x^2 + 12x + 7$ or e.g. $y >$ line

B1ft: Any 2 of $y \leqslant -2x^2 + 12x + 7$, $y \geqslant "\frac{25}{3}"x$, $x \geqslant 0$ with inequalities in the correct directions and ignore any other inequalities for this mark.

Allow strict or non-strict inequalities and condone a mix of both for this mark.

Follow through their equation for $l\left(y = \frac{c}{-b}\right)$ or their coordinates from part (b) and their possibly incorrect completed square form for C.

Examples:

$$y \leqslant -2x^2 + 12x + 7, \quad y \geqslant "\frac{25}{3}"x$$

$$"\frac{25}{3}"x < y < -2x^2 + 12x + 7$$

$$y \leqslant -2x^2 + 12x + 7,\ x > 0$$

For the restriction on x, allow $0 < x < a$ where $a \geqslant "3"$

B1ft: Correct and fully **consistent** inequalities and no incorrect inequalities.

Follow through their equation for $l\left(y = \frac{c}{-b}\right)$ or their coordinates from part (b) and their possibly incorrect completed square form for C.

For the restriction on x, allow $0 < x < a$ where $a \geqslant "3"$ but must be consistent with the other inequalities i.e. strict or non-strict and must not contradict the inequalities given for the line and the curve if "3" is the upper limit.

e.g. $"\frac{25}{3}x" \leqslant y \leqslant -2x^2 + 12x + 7, \quad 0 \leqslant x < 3$ scores B1ftB0ft

If extra inequalities are given that do not contradict then they can be ignored (see examples below)

<u>Examples of full marks in (c):</u>

$$y \leqslant -2x^2 + 12x + 7, \quad y \geqslant "\frac{25}{3}x", \quad x \geqslant 0$$

$$"\frac{25}{3}x" < y < -2x^2 + 12x + 7, \quad 0 < x < 3$$

$$"\frac{25}{3}x" < y,\ y < -2x^2 + 12x + 7, \quad 0 < x,\ y > 0$$

Question Number	Scheme	Marks
6(a)	States or implies that $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)$	M1
	$-12=k\left(2\left(0\right)+3\right)^2\left(0-4\right)\Rightarrow k=...$	**d**M1
	$\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(2x+3\right)^2\left(x-4\right)$ oe e.g. $\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(4x^3-4x^2-39x-36\right)$	A1
		(3)

Notes

In part (a) "f(*x*) =" is not required so just look for the correct expression.

The correct <u>expression</u> must be seen not just e.g. $\left(\mathrm{f}(x)=\right)k\left(2x+3\right)^2\left(x-4\right)$ followed by $k=\frac{1}{3}$ but allow if the correct expression subsequently appears in part (b) provided they have not given an incorrect expression in part (a).

A fully correct expression in (a) with no working scores M1dM1A1

M1: States or implies that $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)$ o.e. such as $\left(\mathrm{f}(x)=\right)\beta\left(x\pm\frac{3}{2}\right)^2\left(x\pm4\right)$

Allow β or k to be any non-zero constant including 1 and could be any letter.

You can ignore a spurious "= 0" e.g. $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)=0$ or e.g. $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)=-12$ for this mark.

Note that e.g. $\left(\mathrm{f}(x)=\right)k\left(3\pm2x\right)^2\left(4\pm x\right)$ or e.g. $\left(\mathrm{f}(x)=\right)\beta\left(\frac{3}{2}\pm x\right)^2\left(4\pm x\right)$ are also acceptable for this mark.

dM1: Substitutes $x = 0$ and $y = -12$ into $\left(\mathrm{f}(x)=\right)k\left(2x\pm3\right)^2\left(x\pm4\right)$ to find a value for k.

Alternatively substitutes $x = 0$ and $y = -12$ into $\left(\mathrm{f}(x)=\right)\beta\left(x\pm\frac{3}{2}\right)^2\left(x\pm4\right)$ to find a value for β.

If no substitution is seen, the coefficient must be correct for their function.

Note that e.g. $\left(\mathrm{f}(x)=\right)k\left(3\pm2x\right)^2\left(4\pm x\right)$ or e.g. $\left(\mathrm{f}(x)=\right)\beta\left(\frac{3}{2}\pm x\right)^2\left(4\pm x\right)$ are also acceptable for this mark.

Depends on the first method mark.

A1: $\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(2x+3\right)^2\left(x-4\right)$ o.e. e.g. $\left(\mathrm{f}(x)=\right)\frac{4}{3}\left(x+\frac{3}{2}\right)^2\left(x-4\right)$ or $\left(\mathrm{f}(x)=\right)-\frac{1}{3}\left(2x+3\right)^2\left(4-x\right)$

Condone a spurious "= 0" e.g. $\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(2x+3\right)^2\left(x-4\right)=0$

Apply isw once a correct expression is seen.

Note that if they start with a **correct** expression e.g. $\left(\mathrm{f}(x)=\right)k\left(2x+3\right)^2\left(x-4\right)$ and expand at the dM1 stage and correctly reach e.g. $-36k=-12\rightarrow k=\frac{1}{3}\rightarrow\left(\mathrm{f}(x)=\right)\frac{1}{3}\left(2x+3\right)^2\left(x-4\right)$ then any errors in the non-constant terms can be ignored and full marks awarded if correct.

	(a) Alternative by simultaneous equations:	
	$(\mathrm{f}(x)=)ax^3+bx^2+cx+d$ $(0,\ -12)\rightarrow d=-12$ $\left(-\frac{3}{2},\ 0\right)\rightarrow 0=-\frac{27}{8}a+\frac{9}{4}b-\frac{3}{2}c-12$ $(4,\ 0)\rightarrow 0=64a+16b+4c-12$ $\mathrm{f}'(x)=3ax^2+2bx+c\rightarrow 0=\frac{27}{4}a-3b+c$	M1
	$a=\frac{4}{3},\ b=-\frac{4}{3},\ c=-13\rightarrow(\mathrm{f}(x)=)\frac{4}{3}x^3-\frac{4}{3}x^2-13x-12$	**d**M1A1

Notes

M1: Starts with a general cubic and uses the given information to produce 3 equations in 3 unknowns with $d=\pm 12$

dM1: Solves to find all unknowns. You do not need to check the working as long as all values are found. They may be found using a calculator.

A1: $(\mathrm{f}(x)=)\frac{4}{3}x^3-\frac{4}{3}x^2-13x-12$ oe. Condone a spurious "= 0" e.g. $(\mathrm{f}(x)=)\frac{4}{3}x^3-\frac{4}{3}x^2-13x-12=0$

Apply isw once a correct answer is seen.

(b)	$\frac{1}{3}(2x+3)^2(x-4)=2x-12 \Rightarrow (4x^2+12x+9)(x-4)=6x-36$ $\Rightarrow 4x^3-16x^2+12x^2-48x+9x-36=6x-36$ $\Rightarrow 4x^3-4x^2=45x$ or e.g. $4x^3-4x^2-45x=0$	M1A1
	$x(4x^2-4x-45)=0 \Rightarrow x=\frac{4\pm\sqrt{736}}{8}$	**d**M1
	$=\frac{1\pm\sqrt{46}}{2}$	A1
		(4)
		(7 marks)

(b) Notes

(b)

M1: Sets their $f(x)$ **which must be a cubic function** equal to $2x-12$ and starts to simplify by expanding the lhs and collecting terms on lhs or from rhs to lhs.
Note that the lhs may already be expanded e.g. if the alternative was used in part (a).

A1: Correct simplified 3 term equation $4x^3-4x^2-45x=0$ or $4x^2-4x-45=0$
oe e.g. $\frac{4}{3}x^3-\frac{4}{3}x^2-15x=0$ or $\frac{4}{3}x^2-\frac{4}{3}x-15=0$
The terms do not necessarily need to be all on one side e.g. $\frac{4}{3}x^2-\frac{4}{3}x=15$ is acceptable.
The "= 0" may be implied by their attempt to solve.

dM1: Solves the resulting 3 term equation which must either be a 3TQ or a 3 term cubic equation with no constant term.
The solution of their equation can be by any method including a calculator.
If values are just written down you may need to check.
Note that it is acceptable to go from e.g. $4x^3-4x^2-45x=0$ to the correct roots using a calculator.
Note that a cubic equation of the required form may have been obtained fortuitously but this mark is still available e.g. if they obtained $(f(x)=)(2x+3)^2(x-4)+24$ in part (a).

A1: Correct values $\frac{1\pm\sqrt{46}}{2}$ oe e.g. $\frac{1}{2}\pm\frac{\sqrt{46}}{2}$
Condone the inclusion of $x = 0$
Ignore any attempts to find the y values.