Question Number	Scheme	Marks
9(a)	$\frac{11\pi}{8}$	B1
		(1)
(b)	102	B1
	States $50\times2+2$ or other suitable reason	**d**B1
		(2)
(c)	$\left(\frac{3\pi}{8}, -5\right)$	B1, B1
		(2)
		(5 marks)

Notes

Do <u>not</u> allow the use of degrees in this question.

If there is more than one answer for any part, score the final attempt.

Remember to check for answers written in the question.

(a)

B1: $\frac{11\pi}{8}$ or exact equivalent. Ignore any reference to the *y* coordinate, correct or incorrect but

e.g. $\left(-1, \frac{11\pi}{8}\right)$ scores B0

(b)

B1: 102

dB1: Gives a suitable reason.

Allow a **correct** mathematical calculation. E.g. $50\times2+2$ or e.g. $50\times2+1+1$ or e.g. $100+2$

If a **correct** mathematical calculation is seen do not be too concerned with the precise explanation for their calculation.

If they state 102 but the reason is **clearly incorrect** e.g. 3 roots between 0 and 2π and another 98 score B1B0

Allow an explanation '2 solutions every 2π radians plus another 2 in the next π radians'

Do not be concerned if it is ambiguous if the boundaries of *x* are included or not.

e.g. "there are 100 solutions for $0 \leqslant x \leqslant 100\pi$ and then another 2 in the next π"

See the next page for a diagram of the scenario.

Depends on the first B mark

(c)

B1: One correct coordinate.

B1: $\left(\frac{3\pi}{8}, -5\right)$

Note that $\left(\text{incorrect value}, -5\right)$ scores B1B0 not B0B1

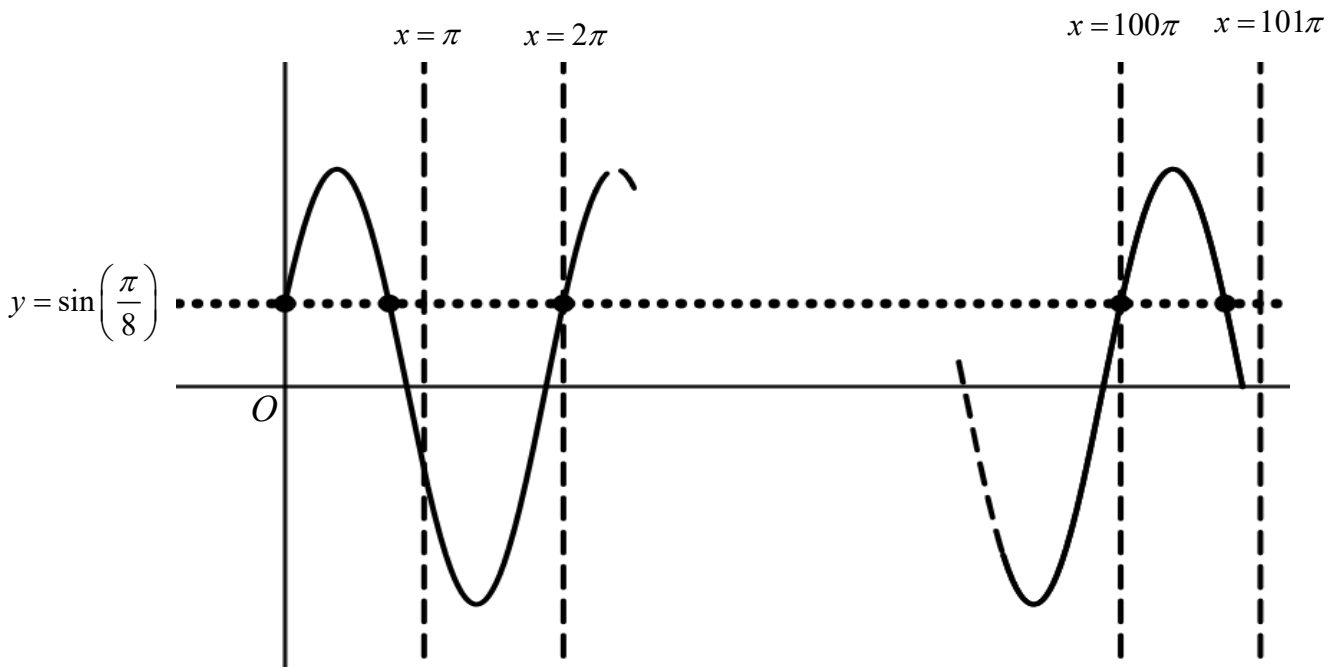
Allow answers to be written as coordinates or as $x = \ldots$, $y = \ldots$

Condone missing brackets e.g. $\frac{3\pi}{8}, -5$ and condone punctuation other than a comma e.g. $\frac{3\pi}{8}; -5$

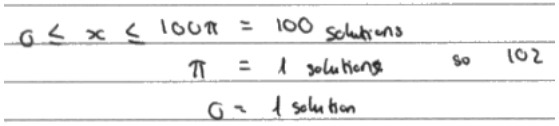
The *x* coordinate must be written as a single fraction e.g. not $\frac{\pi}{2}-\frac{\pi}{8}$

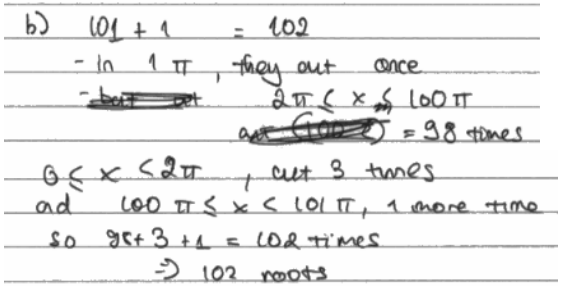
The *y* coordinate must be written as single number so do not allow -5×1 for -5

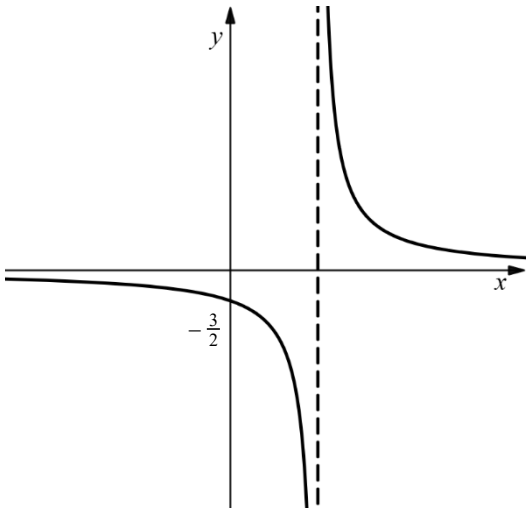
Do **not** allow coordinates the wrong way round e.g. $\left(-5, \frac{3\pi}{8}\right)$ scores B0B0



Note that the following were condoned as acceptable justification for 102 roots:





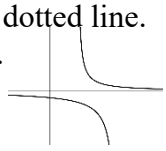
Question Number	Scheme		Marks
10(a)		Shape in quadrant one	M1
		Fully correct shape	A1
		Cuts the y-axis at $-\frac{3}{2}$	B1
			(3)

Notes

Marks cannot be scored without a sketch.

M1: Correct shape in quadrant 1 for the right hand branch. It must not cross either axis but be tolerant of functions that don't go down as far as the x-axis and do extend as far left as the y-axis.
Condone curves that touch the x or y-axis as long as they do not cross the axis.
Ignore any asymptotes, correct or incorrect, for this mark.
Ignore the other branch for this mark.

A1: Correct shape and position for whole curve. Be tolerant of slips of the pen but the intention should be clear. Look for the curve being in quadrants 1, 3 and 4 with a vertical asymptote in quadrant 1 and 4. The asymptote does **not** need to be drawn e.g. as a dotted line.
Do not allow this mark if the branches clearly overlap vertically. E.g.



The left hand branch must extend into quadrant 3 and should not intentionally dip away from the x-axis to e.g. create an intentional maximum.
There should be no horizontal asymptotes other than the x-axis.

B1: Cuts or touches the negative y-axis **once** at $-\frac{3}{2}$. Do **not** allow $-\frac{3k}{2k}$ but $\frac{3}{-2}$ is ok.
Must be in the correct position but allow as $-\frac{3}{2}$ or $y=-\frac{3}{2}$ or $\left(0,-\frac{3}{2}\right)$ and condone $\left(-\frac{3}{2},0\right)$ as long as it is in the correct place. Condone missing brackets e.g. $0,-\frac{3}{2}$
Allow to appear away from the sketch but it must be fully correct e.g. $\left(0,-\frac{3}{2}\right)$
If there is any ambiguity, the sketch takes precedence.

Note that there is no requirement in the question to label the asymptote so any labels, correct or incorrect can be ignored.

(b)	$\frac{3k}{x-2k}=12-x \Rightarrow 3k=12x-x^2-24k+2kx$	M1 A1
	$b^2-4ac=\left(12+2k\right)^2-4\times1\times27k$	**d**M1
	$\Rightarrow 144+48k+4k^2-108k=0 \Rightarrow k^2-15k+36=0$ *	A1*
		(4)

Notes

(b)

M1: Equates equations for curve with given line and attempts to form a quadratic equation in x. The terms do not need to be collected.

Look for $\frac{3k}{x-2k}=12-x$ (condoning slips), cross multiplies and proceeds to a quadratic equation which may be implied by an attempt at the discriminant.

Condone invisible brackets e.g. $\frac{3k}{x-2k}=12-x \rightarrow 3k=12-x\left(x-2k\right)$ as long as a quadratic equation in x is obtained and allow full recovery for the subsequent marks.

A1: Obtains a correct quadratic equation with brackets expanded which may be unsimplified e.g. $\frac{3k}{x-2k}=12-x \Rightarrow 3k=12x-x^2-24k+2kx$. May be implied.

dM1: Attempts b^2-4ac for their $ax^2+bx+c=0$ with both b and c expressions in terms of k. Condone slips/miscopies but they must be attempting the discriminant "correctly" e.g. if they expand to $3k=12x-x^2-24k+2kx$, they can't just find the discriminant of the rhs. May be seen embedded in an attempt at the quadratic formula.

A1*: Sets $b^2-4ac=0$ oe e.g. $b^2=4ac$, expands and simplifies to obtain $k^2-15k+36=0$ from **fully correct work**. Note that there must be at least one intermediate line between e.g. $\left(12+2k\right)^2-4\times1\times27k=0$ and $k^2-15k+36=0$

Note that the "= 0" must appear before the printed answer but may be as a preamble e.g. $b^2-4ac=0$ etc.

There must have been no incorrect statement within their solution e.g. $b^2-4ac<0$

<u>Note that is also possible to answer (b) in terms of y and this marks in the same way:</u>

$\frac{3k}{12-y-2k}=y \Rightarrow 3k=12y-y^2-2ky$ M1A1

$b^2-4ac=\left(2k-12\right)^2-4\times1\times3k$ **d**M1

$4k^2-48k+144-12k=0 \Rightarrow k^2-15k+36=0$ * A1

(c)	$k^2 - 15k + 36 = 0 \Rightarrow (k =) 3, 12$	B1
	$k = 3 \Rightarrow x^2 - 18x + 81 = 0 \left(\text{or just } x = \frac{18}{2} \right) \Rightarrow x = 9 \Rightarrow y = 3$ or $k = 12 \Rightarrow x^2 - 36x + 324 = 0 \left(\text{or just } x = \frac{36}{2} \right) \Rightarrow x = 18 \Rightarrow y = -6$	M1 A1
	$P = (18, -6)$	A1
		(4)

Notes

(c)

B1: Correct values for k. May be seen anywhere.
No method is required so just look for the correct values.
"$k =$" is not required and condone e.g. $x = 3, 12$

M1: Substitutes either of their values for k into their $x^2 - (12 + 2k)x + 27k = 0$ to set up an equation in x and solves to find a value for x **and** a corresponding value for y.

Note that as the quadratic in x has equal roots, the root may be found from $x = \frac{-b}{2a}$ so that the 3TQ in x may not be seen.

An alternative would be to solve C and l simultaneously with either of their values of k to find x or y via a 3TQ **and then find the value of the other variable**.

e.g $k = 3 \Rightarrow \frac{9}{x-6} = 12 - x \Rightarrow x^2 - 18x + 81 = 0 \Rightarrow x = ...(9) \Rightarrow y = ...(3)$

or e.g. $k = 3 \Rightarrow y = \frac{9}{12 - y - 6} \Rightarrow y^2 - 6y + 9 = 0 \Rightarrow y = ...(3) \Rightarrow x = ...(9)$

or

$k = 12 \Rightarrow \frac{36}{x-24} = 12 - x \Rightarrow x^2 - 36x + 324 = 0 \Rightarrow x = ...(18) \Rightarrow y = ...(-6)$

or e.g. $k = 12 \Rightarrow y = \frac{36}{12 - y - 24} = 12 - x \Rightarrow y^2 + 12y + 36 = 0 \Rightarrow y = ...(-6) \Rightarrow x = ...(18)$

In the above examples, the "= 0" in e.g. $x^2 - 18x + 81 = 0$ may be implied by their attempt to solve.

May be implied by correct or correct ft values of x and y.

A1: Correctly finds either $x = 9, y = 3$ **or** $x = 18, y = -6$

A1: Coordinates of P are $x = 18, y = -6$

Must clearly be identified as the only solution.
Allow answers to be written as coordinates or as $x = \ldots, y = \ldots$
Condone missing brackets e.g. $18, -6$ and condone punctuation other than a comma e.g. $18; -6$

		(11 marks)