7. The curve C has equation

$$y = \frac{2}{3}x^3 - 8x^2 + 43x - \frac{20}{3}$$

(a) Show that $\frac{dy}{dx}$ can be written in the form

$$p(x + q)^2 + r$$

where p, q and r are constants to be found.

(5)

(b) Hence state

(i) the minimum value of $\frac{dy}{dx}$

(ii) the value of x at which this minimum value occurs.

(2)

Given that S is the point on C at which the gradient is a minimum,

(c) find the equation of the tangent to C at S, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

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Question 7 continued

(Total for Question 7 is 10 marks)

8. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = f(x), \quad x > 0$

A point P lies on C.

Given that

- $f'(x) = 4x^2 + \frac{6}{x^2} - 4$
- the equation of the tangent to C at P is $y = 10x - 6\sqrt{3}$

(a) (i) verify that $\sqrt{3}$ is a possible x coordinate of P,

(ii) find, using algebra, the other possible x coordinate of P.

(6)

Given that the x coordinate of P is $\sqrt{3}$

(b) find an equation of the normal to C at P.

(2)

(c) Find $f(x)$, writing your answer in simplest form.
You must show each stage of your working.

(4)

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