6.

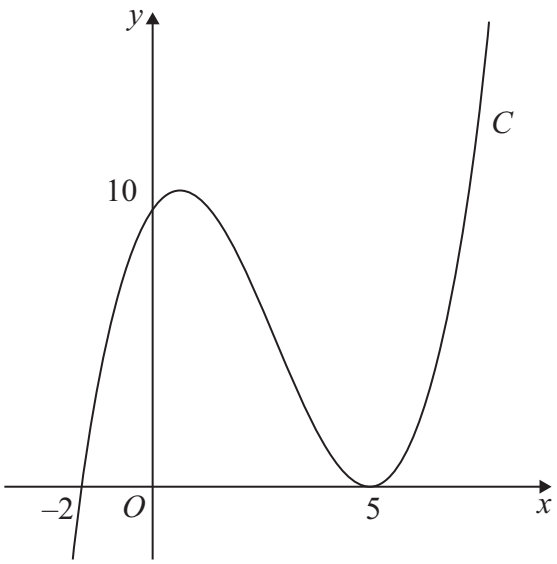

Figure 3

Figure 3 shows a sketch of the curve C with equation $y = f(x)$ where $f(x)$ is a cubic function in x.

The curve C

- cuts the x-axis at $(–2, 0)$ and cuts the y-axis at $(0, 10)$
- touches the x-axis at $(5, 0)$

as shown in Figure 3.

(a) Deduce the roots of the equation

(i) $f\left(\frac{1}{3}x\right) = 0$

(ii) $f(x – 3) = 0$

(2)

(b) Find an expression for $f(x)$. You should leave your answer in factorised form.

(3)

The curve C intersects the straight line $y = 10(x + 2)$ at exactly three points.

(c) Use algebra to find the exact x coordinates of the three points of intersection.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 9 marks)

7. The curve C has equation

$$y = \frac{2}{3}x^3 - 8x^2 + 43x - \frac{20}{3}$$

(a) Show that $\frac{dy}{dx}$ can be written in the form

$$p(x + q)^2 + r$$

where p, q and r are constants to be found.

(5)

(b) Hence state

(i) the minimum value of $\frac{dy}{dx}$

(ii) the value of x at which this minimum value occurs.

(2)

Given that S is the point on C at which the gradient is a minimum,

(c) find the equation of the tangent to C at S, giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

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