| Question Number | Scheme | Marks |
|---|---|---|
| **4(i)** | Uses a correct law of indices on $2^{4k-3}$ or $8^{1-k}$<br>The possibilities are endless but some more common examples are:<br>For $2^{4k-3}$: $2^{4k}\times 2^{-3}$, $\dfrac{2^{4k}}{8}$, $8^{\frac{4k-3}{3}}$, $4^{\frac{4k-3}{2}}$, $\left(\sqrt{2}\right)^{8k-6}$<br>For $8^{1-k}$: $8\times 8^{-k}$, $\dfrac{8}{8^{k}}$, $2^{3(1-k)}$, $4^{\frac{2(1-k)}{3}}$, $\left(\sqrt{2}\right)^{6-6k}$<br>These may be seen in isolation e.g. not in an equation.<br>But not just e.g. $8=2^3$ | M1 |
| | e.g.<br>$2^{4k-3}=\dfrac{8^{1-k}}{4\sqrt{2}}\Rightarrow 2^{4k-3}=\dfrac{2^{3(1-k)}}{2^{\frac{5}{2}}}\Rightarrow 2^{4k-3}=2^{3(1-k)-\frac{5}{2}}$<br>$\Rightarrow 4k-3=3(1-k)-\dfrac{5}{2}\Rightarrow k=...$ | dM1 |
| | $k=\dfrac{1}{2}$ | A1 |
| | | **(3)** |

**Notes**

**Part (i) can be solved using many different approaches.**
**Please follow the general guidelines below and seek advice from your Team Leader if necessary.**
**Note that it is acceptable to use logs within their solution as long as the first mark is scored but they should have a base that is a power of 2 (see note below*)**

**(i)**

**M1:** This is effectively a B mark and is for applying a correct law of indices on $2^{4k-3}$ or $8^{1-k}$
See scheme for some examples.

Note that this may be implied by e.g. $x=2^{k}\Rightarrow 2^{4k-3}=\dfrac{x^{4}}{8}$

**dM1:** For this mark they must
- reach a value for *k*
- reach an equation of a form so that the indices in terms of *k* can be compared e.g. $\alpha^{f(k)}=\alpha^{g(k)}\Rightarrow f(k)=g(k)$ **or** of the form $\alpha^{f(k)}=\beta$ where $f(k)$ and $g(k)$ are linear

Note that the second bullet point may be implied by e.g. $2^{4k-3}=\dfrac{2^{3(1-k)}}{2^{\frac{5}{2}}}\Rightarrow 4k-3=3(1-k)-\dfrac{5}{2}$

**A1:** Obtains $k=\dfrac{1}{2}$ from a correct equation of one of the forms above.

Note that correct work leading to $128^{k}=8\sqrt{2}\Rightarrow k=\dfrac{1}{2}$ or e.g. $k=\log_{128}8\sqrt{2}=\dfrac{1}{2}$ is acceptable.

*But note that e.g. $7k=\dfrac{\log_{10}8\sqrt{2}}{\log_{10}2}\Rightarrow k=\dfrac{1}{2}$ is not acceptable without justification and scores a maximum of M1dM1A0

**Correct answers with no working in (i) score no marks.**

| | | |
|---|---|---|
| **(ii)** | $\dfrac{x\sqrt{3}+2}{\sqrt{3}-1}=x\sqrt{3}-4\Rightarrow x\sqrt{3}+2=\left(x\sqrt{3}-4\right)\left(\sqrt{3}-1\right)$<br>$\Rightarrow x\sqrt{3}+2=3x+4-4\sqrt{3}-x\sqrt{3}$<br>$\Rightarrow 2\sqrt{3}x-3x=2-4\sqrt{3}$ | M1 |
| | $\Rightarrow x\left(2\sqrt{3}-3\right)=2-4\sqrt{3}$ | A1 |
| | $\Rightarrow x=\dfrac{2-4\sqrt{3}}{2\sqrt{3}-3}\times\dfrac{2\sqrt{3}+3}{2\sqrt{3}+3}$ | dM1 |
| | $=-6-\dfrac{8}{3}\sqrt{3}$ | A1 |
| | | **(4)** |

| | | |
|---|---|---|
| | **Alternative** | |
| | $\dfrac{x\sqrt{3}+2}{\sqrt{3}-1}=x\sqrt{3}-4\Rightarrow \dfrac{x\sqrt{3}+2}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1}=x\sqrt{3}-4$<br>$\Rightarrow \dfrac{3x+2\sqrt{3}+x\sqrt{3}+2}{3-1}=x\sqrt{3}-4$<br>$\Rightarrow 3x+2\sqrt{3}+x\sqrt{3}+2=2x\sqrt{3}-8\Rightarrow \sqrt{3}x-3x=10+2\sqrt{3}$ | M1 |
| | $\Rightarrow x\left(\sqrt{3}-3\right)=10+2\sqrt{3}$ | A1 |
| | $x=\dfrac{10+2\sqrt{3}}{\sqrt{3}-3}\times\dfrac{\sqrt{3}+3}{\sqrt{3}+3}$ | dM1 |
| | $=\dfrac{10\sqrt{3}+30+6+6\sqrt{3}}{3-9}=-6-\dfrac{8}{3}\sqrt{3}$ | A1 |

**(ii)**

**M1:** Attempts to form a linear equation in $x$ with $x$ terms on one side and constants on the other. The $x$ does not need to be factorised out from the $x$ terms for this mark.

**A1:** A **correct** linear equation in $x$ with terms collected and $x$ factorised out.

Possible correct equations are: $x\left(2\sqrt{3}-3\right)=2-4\sqrt{3}$ or $x\left(\sqrt{3}-3\right)=10+2\sqrt{3}$

May be unsimplified e.g. $x\left(\sqrt{3}+\sqrt{3}-3\right)=4-2-4\sqrt{3}$

**dM1:** Makes $x$ the subject and correctly rationalises the denominator of $x$ of the form $\alpha+\beta\sqrt{3}$ where $\alpha$ and $\beta$ are non-zero.

e.g. $x=\dfrac{...}{\alpha+\beta\sqrt{3}}\times\dfrac{\alpha-\beta\sqrt{3}}{\alpha-\beta\sqrt{3}}$ oe and proceeds to an answer of the form $a+b\sqrt{3}$ or $\dfrac{a+b\sqrt{3}}{c}$

Sight of e.g. $x=\dfrac{2-4\sqrt{3}}{2\sqrt{3}-3}\times\dfrac{2\sqrt{3}+3}{2\sqrt{3}+3}$ is sufficient but may be implied by e.g.

$x=\dfrac{2-4\sqrt{3}}{2\sqrt{3}-3}=\dfrac{4\sqrt{3}+6-24-12\sqrt{3}}{12-9}$

**A1:** Reaches $-6-\dfrac{8}{3}\sqrt{3}$ following dM1

Allow equivalents e.g. $-\dfrac{36}{6}-\dfrac{16}{6}\sqrt{3}$ but not e.g. $\dfrac{-18-8\sqrt{3}}{3}$

Apply isw once the correct answer is seen.

**(ii) Alternative method: Attempts to rationalise first**

**M1:** Multiplies the left hand side of the equation by $\frac{\sqrt{3}+1}{\sqrt{3}+1}$ leaving the rhs unchanged and then attempts to form a linear equation in $x$ with $x$ terms on one side and constants on the other. The $x$ does not need to be factorised out from the $x$ terms for this mark.

Condone invisible brackets even if not recovered e.g.

$$\frac{x\sqrt{3}+2}{\sqrt{3}-1}=x\sqrt{3}-4 \Rightarrow \frac{x\sqrt{3}+2\left(\sqrt{3}+1\right)}{3-1}=x\sqrt{3}-4$$

**A1:** A **correct** linear equation in $x$ with terms collected and $x$ factorised out.

**dM1:** Makes $x$ the subject and correctly rationalises the denominator of $x$ of the form $\alpha+\beta\sqrt{3}$ where $\alpha$ and $\beta$ are non-zero.

e.g. $x=\frac{...}{\alpha+\beta\sqrt{3}}\times\frac{\alpha-\beta\sqrt{3}}{\alpha-\beta\sqrt{3}}$ oe and proceeds to an answer of the form $a+b\sqrt{3}$.

Sight of e.g. $x=\frac{10+2\sqrt{3}}{\sqrt{3}-3}\times\frac{\sqrt{3}+3}{\sqrt{3}+3}$ is sufficient but may be implied by e.g.

$$x=\frac{10+2\sqrt{3}}{\sqrt{3}-3}=\frac{10\sqrt{3}+30+6+6\sqrt{3}}{3-9}$$

**A1:** Reaches $-6-\frac{8}{3}\sqrt{3}$ following dM1

Allow equivalents e.g. $-\frac{36}{6}-\frac{16}{6}\sqrt{3}$ but not e.g. $\frac{-18-8\sqrt{3}}{3}$

Apply isw once the correct answer is seen.